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Energy-Momentum Quanta in Fresnel's Evanescent Wave

O. COSTA DE BEAUREGARD

Institut Henri Poincaré, 11 rue Pierre et Marie Curie
75 Paris, France

CH. IMBERT

Institut d'Optique, Faculté des Sciences
91 Orsay, France

and

J. RICARD

Institut Henri Poincaré, 11 rue Pierre et Marie Curie
75 Paris, France

Abstract

Fresnel's theory of the evanescent wave in total reflection entails that the propagation vector **k** and the momentum quanta $\hbar\mathbf{k}$ have an imaginary component and, thus, a projection on the reflecting plane that is larger (in units such that $c = 1$) than the angular frequency ω and the energy quanta $\hbar\omega$. We discuss the 'tachyon properties' of these energy-momentum quanta and propose an experimental test using absorption or stimulated emission by an atomic or ionic beam. We then show that the Maxwell–Minkowski tensor (although certainly appropriate to discuss the macroscopic energy-momentum exchange between wave and diopter) does not describe adequately the energy-momentum density of the quanta in the evanescent wave, this stemming from its too remote connection with the generator ∂_i of space-time displacements. On the other hand de Broglie's energy-momentum tensor $A_k[\partial^i]B^{jk}$ is *the* density canonically associated with the generator of space-time displacements; we show that it describes quite satisfactorily both the energy fluxes (as measured through the longitudinal Goos–Hänchen and our new transverse shifts of the reflected beam in total reflection) and the momentum densities of the quanta inside the evanescent wave. Finally, we show that it is the gauge which is transverse in the diopter's rest frame that directly yields the physically measured energy fluxes. We take this fact as a new argument, strongly supported by experimental evidence, in favour of the physical reality of electromagnetic potentials.

1. *Introduction*

Our interest for the energy-momentum quanta inside Fresnel's evanescent wave was prompted by our previous studies (de Beauregard, 1965; Imbert & Ricard, 1968, 1970; Imbert, 1968; Ricard, 1970) concerning the transverse shift in total reflection of a circularly polarized light beam. The driving idea for undertaking these studies was that, under appropriate circumstances, the velocity and momentum of spinning particles are non-collinear, an idea which de Beauregard (1942) and Weyssenhof (1947) had related to asymmetry of the energy-momentum tensor of spinning media, and which had earlier been proposed in other contexts (Frenkel, 1926; Mathisson, 1931, 1933, 1937; Proca, 1933).

Recent experiments by one of us (Imbert, 1969, 1970a, b) have provided an excellent verification of the formula Imbert (1968) had derived, as a specification of the general theory (de Beauregard, 1965) for the case of total reflection. The new transverse shift was calculated by expressing conservation of the energy flux as represented by the Poynting vector, following a procedure analogous to Kristoffel's (1956) and Renard's (1964) derivation of the longitudinal Goos–Hänchen shift (Goos & Hänchen, 1947). As the Poynting vector is part of the Maxwell–Minkowski energy-momentum tensor, and this tensor is essentially symmetric in the vacuum of the evanescent wave of total reflection, it may seem at first sight that in our recent publications we have departed from our original philosophy (de Beauregard, 1942, 1943). Our answer is that we have constantly kept in mind that in total reflection of a plane (elliptically polarized) wave the component $\hbar k^z$ of the photon's momentum orthogonal to the incidence plane is identically zero (as all the field magnitudes are z independent); thus, while the $M^{4\alpha}$ ($\alpha = 1, 2, 3$) components of the Maxwell–Minkowski tensor give the values of the energy fluxes which have been measured *via* the longitudinal and transverse shifts of the reflected beam, the $M^{z4} = M^{4z}$ component does *not* give the right momentum density, which should be zero in all circumstances. Thus an appropriate energy-momentum tensor for describing the properties of quanta inside the evanescent wave in vacuo *must* be asymmetrical—and thus the Maxwell–Minkowski tensor is not appropriate in this respect.

At this point we decided to investigate the whole picture of energy-momentum quanta inside the evanescent wave, and we noticed immediately that, following Fresnel's well-known theory, the k^y component of the propagation vector **k** normal to the reflecting plane is imaginary, while the k^x component parallel to both the reflecting and the incidence planes is (in units such that $c = 1$) *larger* than the angular frequency ω. Speaking in terms of energy-momentum quanta, and using units such that $c = 1$, this amounts to saying that the imaginary character of one of the components of their momentum entails 'tachyon properties' (de Beauregard, 1970) for the light quanta inside the evanescent wave—very much like what their alleged imaginary proper mass does for tachyons proper.†

Section 2 of this paper is devoted to a discussion of the tachyon properties of the photons inside the evanescent wave, and to possible means

† G. Feinberg [*Physical Review*, **159**, 1089 (1967)] has baptized these hypothetical particles proposed by various authors,the first of which may well have been J. P. Terletsky [*Journal de Physique et le radium*, **21**, 681 (1960)] and S. Tanaka [*Progress of Theoretical Physics*, **24**, 171 (1960)].

of measuring the ratio $k^x/\omega > 1$ in first-order absorptions or stimulated emissions.

Here again the Maxwell–Minkowski tensor turns out to be faulty, as the ratio iM^{x4}/M^{44} (in the metric x, y, z, it) is constant in the evanescent wave with the value $iM^{x4}/M^{44} = \omega/k^x < 1$, that is, exactly the inverse of the value $k^x/\omega > 1$ expected from the consideration of the tachyon photons. The reason for this defect is, just as before, that the Maxwell–Minkowski ($i = 1, 2, 3, 4$) tensor is not *directly* related to the generator ∂_i of space-time translations.

Now, there *is* an energy momentum tensor which is asymmetric in the vacuum and which is *directly* related to the generator ∂_i of space-time displacements: de Broglie's† canonical energy-momentum tensor for the photon field $T^{ij} \equiv A_K[\partial^i]B^{jK}$ [with, as usual, $[\partial^i] \equiv \underset{\rightarrow}{\partial}^i - \underset{\leftarrow}{\partial}^i$, $B^{jK} \equiv \partial^j A^K - \partial^K A^j$, $\partial_i A^i = 0$]. By its very definition, this tensor yields here the correct values $T^{z4} = 0$ and $iT^{x4}/T^{44} = k^x/\omega > 1$. Of course, there remains to be seen if it also yields the correct values $T^{4x} = M^{4x}$ and $T^{4z} = M^{4z}$ that have been measured by means of the longitudinal Goos–Hänchen and the new transverse displacements. The answer is *yes* (Imbert & Ricard, 1968; Ricard, 1970)—provided one uses the gauge that is transverse in the rest frame of the refracting medium. So (*modulo* the latter condition) everything is settled concerning both the energy flux and the momentum density if one takes the asymmetric de Broglie tensor to be the energy-momentum density of the photons inside the evanescent wave of total reflection.

Then, of course, one must ask what happens with the other gauges. The answer is that, contrary to what occurs with ordinary plane waves, the de Broglie tensor is *not* gauge invariant in Fresnel's evanescent wave, and that, things being so, the experimental values of the energy fluxes iT^{4x} and iT^{4z} (as measured through the longitudinal and transverse shifts of the reflected beam) unequivocally select the (locally) transverse gauge as *the* good one. This argument belongs to the same family as de Broglie's (1947, 1950) one pertaining to potential energy and the mass defect (Brillouin, 1964), but it seems to us to be much more compulsory. It is in some sense symmetrical to the Aharonov–Bohm (Aharonov & Bohm, 1959) type of argument (Ehrenberg & Siday, 1949) [also supported by experimental verifications (Chambers, 1960; Boersch, 1962)],‡ as it fixes the gauge in a situation where fields are strongly present, while the Aharonov–Bohm one shows an influence of the potentials in a region where no field is present, but without fixing the gauge.

† de Broglie, L. (1949). *Mécanique Ondulatoire du Photon et Théorie Quantique des Champs*, p. 43. Gauthier-Villars, Paris. From the Lagrangian $L \equiv A_i[\partial_j]B^{ji} + \frac{1}{2}B_{ij}B^{ij}$, one deduces the field equations $B_{ij} = \partial_i A_j - \partial_j A_i$, $\partial_j B^{ji} = 0$ and the expression of the canonical energy momentum tensor $A_k(\partial^i)B^{jk}$.

‡ See also, Jaklevic, R. G., Lambe, J. J., Silver, A. M. and Mercereau, J. E. (1964). *Physical Review Letters*, **12**, 274. In this experiment in solid state physics the spatial domains where the magnetic field and the electric current are non-zero are completely distinct.

We discuss in Sections 3 and 4 the properties of the Maxwell–Minkoswki and the de Broglie energy-momentum tensors inside Fresnel's evanescent wave.

2. *Tachyon Photons*

We shall use for simplicity units such that $c = 1$ and $\hbar = 1$.

A plane monochromatic wave of angular frequency ω travelling through a medium of index n with a propagation vector $\mathbf{k}_i$, $k_i = n\omega > \omega$ of components $k_i^x = n\alpha\omega > \omega$, $k_i^y = n\beta\omega > 0$, $k_i^z = 0$ (Fig. 1), and undergoing

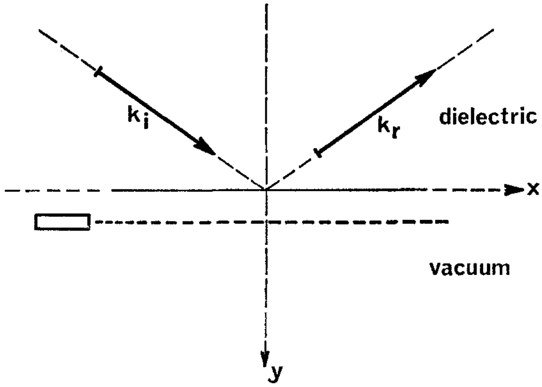

Figure 1.—Ionic beam undergoing transitions with tachyon-photons inside Fresnel's evanescent wave.

total reflection on vacuum on a plane $y = 0$, generates an evanescent wave obeying the same formulas as an ordinary plane wave but with a complex propagation vector $\mathbf{k}$ of components

$$k^x = k_i^x = n\alpha\omega > \omega, \qquad k^y = -j(n^2\alpha^2 - 1)^{1/2}\,\omega, \qquad k^z = 0.$$

Thus the momentum quanta $\mathbf{k}$ associated with the energy quanta ω in the wave are complex, with an x component larger than ω and an imaginary y component. As appropriate experiments have displayed the absorption of the energy quanta ω in the evanescent wave, there is no reason why the absorption of the momentum quanta k^x (which enter the phase in exactly the same fashion, and are connected with ω in a Lorentz transformation) should not be detectable also. It is then clear that, in simultaneous absorption (or stimulated emission) of the energy and momentum quanta ω and k^x, the photons of the evanescent wave will display a tachyon-like property, their imaginary momentum component k^y entailing consequences similar (de Beauregard, 1970) to those of the (hypothetic) imaginary proper mass of tachyons proper.†

Now we must examine how the imaginary momentum quantum k^y will behave in absorption or stimulated emission. Suppose (Fig. 1) that a beam

† See footnote † on page 127.

of particles travels *in vacuo* parallel, and very close to, the plane interface $y = 0$, and that it is expressed in the form of a Fourier expansion

$$\psi = \exp[j(\Omega t - K^x x)] \int \exp(jK^y y)\, \xi(K^y)\, dK^y \tag{2.1}$$

where $|\xi| \simeq 0$ except for $K^y \simeq 0$.

The contribution of the imaginary part of the photon's phase can also be Fourier expanded, so that the Fresnel evanescent wave assumes the expression

$$\mathbf{A} = \exp[j\omega(t - n\alpha x)] \int \exp[(jk_0{}^y y)\, \mathbf{B}(k_0{}^y)\, dk_0{}^y] \tag{2.2}$$

But, as the energy-momentum relation

$$\omega^2 = (k^x)^2 + (k^y)^2 \tag{2.3}$$

must be satisfied and the values of ω and $k^x > \omega$ are imposed, no real value $k_0{}^y$ will satisfy (2.3). Thus only the whole phase coherent Fourier integral (2.2) can be absorbed or emitted; that is, in the Feynman-style

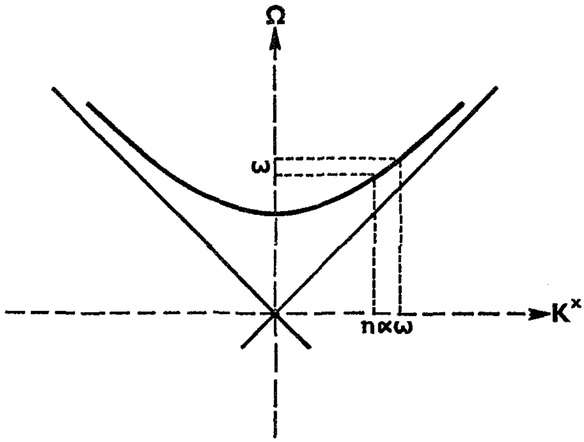

Figure 2.—Free electrons undergoing a first-order photoelectric effect with tachyon-photons inside Fresnel's evanescent wave.

formulas for the transition amplitudes, the imaginary k^y should be inserted. Finally, the y expression of the transition amplitude will simply be the product of the two amplitudes $\psi(y)$ and $A(y) = A_0 \exp[-(n^2\alpha^2 - 1)^{1/2} \omega y]$, so that if the ψ beam is, say, a Gaussian distribution centered on y_0 and very narrow with respect to the penetration depth $1/(n^2\alpha^2 - 1)^{1/2}\omega$ of the evanescent wave, it will simply feel a plane tachyon wave with $k^x = n\alpha\omega > \omega$ and amplitude $A(y_0)$.

Such circumstances should allow remarkable experiments, the prototype of which would be a first-order photoelectric effect on free electrons. This would occur (Fig. 2) if

$$dK^x/d\Omega = n\alpha \tag{2.4}$$

that is, according to the formula

$$\Omega^2 - (K^x)^2 = m^2 \tag{2.5}$$

where m denotes the electron's rest mass, if

$$n\alpha K^x = \Omega \equiv E + m \tag{2.6}$$

where E denotes the kinetic energy, or

$$E = \{1/\sqrt{(1 - 1/n^2\alpha^2)} - 1\}m \tag{2.7}$$

However, this very striking experiment would be an exceedingly difficult one, because it is impossible to have both large energy quanta ω and a large penetration depth $1/(n^2\alpha^2 - 1)^{1/2}\omega$ of the evanescent wave. But, very fortunately, other possibilities exist.

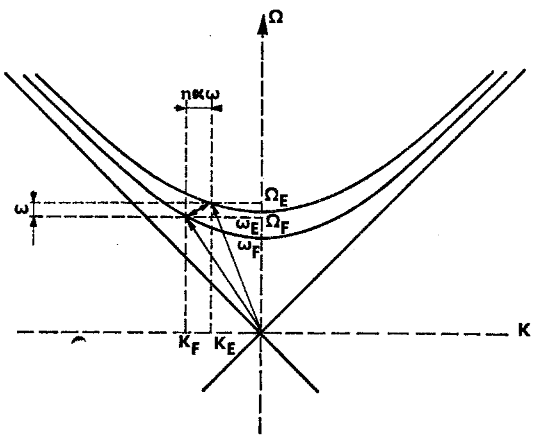

Figure 3.—Imparting to the non-excited (resp., excited) ions the right momentum $\hbar K_F$ (resp., $\hbar K_E$), the absorption (resp., emission) transition with tachyon-photons is rendered possible: $\omega = \Omega_E - \Omega_F < \omega_E - \omega_F$.

By choosing the photon wave in, say, the centimetric Hertzian range, we can obtain a deep penetration of the evanescent wave, and nevertheless detect the energy quanta ω by absorbing them in a first-order transition $\Omega_F \rightarrow \Omega_E$. Denoting ω_F and ω_E the rest masses of the states F and E, the transition will be possible only (Fig. 3) if we give the particles exactly the right velocity β_F such that the vector $u^i(i = 1, 4; x^4 = it)$ of components $u^x = n\alpha\omega$, $u^4 = i\omega$, joins a point on the hyperbola

$$\Omega_F{}^2 - K_F{}^2 = \omega_F{}^2 \tag{2.8a}$$

to a point on the hyperbola

$$\Omega_E{}^2 - K_E{}^2 = \omega_E{}^2 \tag{2.8b}$$

This will display the simultaneous absorption of the energy and the (real) momentum quanta of the tachyon-photons.

The corresponding velocity β_F (or equivalently the β_E corresponding to stimulated emission) is easily calculated from (2.8a) and (2.8b)

$$K_F = \beta_F \Omega_F, \qquad K_E = \beta_E \Omega_E \tag{2.9}$$

and

$$\Omega_E - \Omega_F = \omega, \qquad K_E - K_F = n\alpha\omega \tag{2.10}$$

Solving these equations amounts to calculate the coordinates of the intersection points of the hyperbola (E) [resp. (F)] with the hyperbola (F) [resp. (E)] translated by u^i(resp. $-u^i$). Two of the four intersection points are at infinity, the other two corresponding to the equation for β_F (or a similar one for β_E)

$$\{(2n\alpha\omega_F\,\omega)^2 + |\ |^2\}\beta_F{}^2 - 8n\alpha\omega_F{}^2\,\omega^2\,\beta_F + \{(2\omega_F\,\omega)^2 - |\ |^2\} = 0 \tag{2.11}$$

with

$$|\ | \equiv |\omega_E{}^2 - \omega_F{}^2 + (n^2\,\alpha^2 - 1)\,\omega^2| \tag{2.12}$$

The small root (in absolute value) is

$$\beta_F = \frac{4n\alpha\omega_F{}^2\,\omega^2 - |\ |\,\{|\ |^2 + 4(n^2\,\alpha^2 - 1)\,\omega_F{}^2\,\omega^2\}^{1/2}}{(2n\alpha\omega_F\,\omega)^2 + |\ |^2} \tag{2.13a}$$

(the corresponding expression for β_E being obtained by the substitution $F \to E$ which leaves $|\ |$ invariant).

In fact, $\epsilon \equiv \omega_E - \omega_F$ and ω being much smaller than ω_F, an approximate expression for (2.12) is $2\omega_F\epsilon$ and for (2.13a) (the distinction between β_F and β_E being thus lost)

$$\beta_F \simeq \frac{n\alpha\omega^2 - \epsilon[\epsilon^2 + (n^2\,\alpha^2 - 1)\,\omega^2]^{1/2}}{n^2\,\alpha^2\,\omega^2 + \epsilon^2} \tag{2.13b}$$

Defining $\Delta\epsilon \equiv \epsilon - \omega$, $\Delta\epsilon \ll \epsilon$, ω, after an easy calculation, we obtain

$$\frac{\Delta\epsilon}{\epsilon} = n\alpha\beta \tag{2.13c}$$

that is, the formula of a generalised Doppler shift with $n\alpha > 1$ (for $n\alpha \leqslant 1$, the formula would be that of the ordinary Doppler shift of the refracted wave). Thus the x component of the evanescent wave's group velocity is $n\alpha > 1$—again a tachyon property. For the transition to be detectable it is necessary that η is much larger than the line breadths of both frequencies ω and ϵ; as in fact $n\alpha$ is not very different from 1, let us say that we should have $\eta/\epsilon \simeq \frac{1}{2}\,10^{-3}$ or $\beta_F \simeq \frac{1}{2}\,10^{-3}$. The kinetic energy corresponding to that velocity and a molecular mass $M \simeq 18.1840.5.10^5$ eV$=1$, 6.10^{10} eV is $M\beta^2/2 \simeq 2{\cdot}000$ eV, a value easily attained with an $N_{15}H_3$ ionic beam.

The 'tachyon' character of the absorbed (or emitted) photons is, of course, expressed by the space-like character of the vector $(n\alpha\omega, i\omega)$ (Fig. 4). Thus, in the reference frame of velocity $\beta_0 = 1/n\alpha$ with respect to the medium, the tachyon photons have no energy, but only a momentum; and, in reference frames of velocity $1/n\alpha < \beta < 1$, they have a 'negative energy'; that is, emission and absorption processes are exchanged, and (due to its

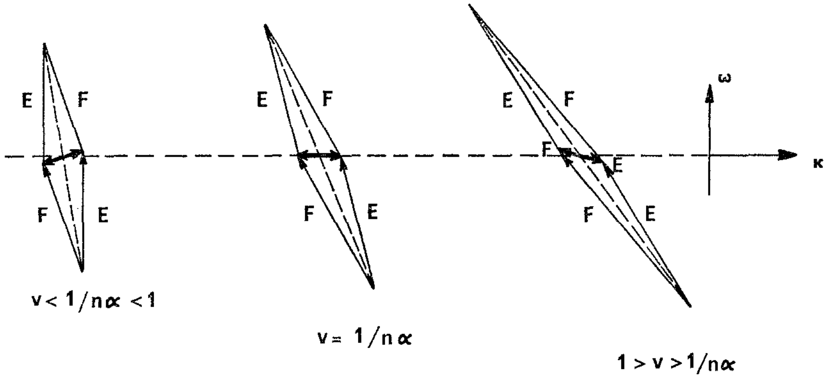

Figure 4.—In reference frames of velocity β^x smaller than, equal to, or larger than $\beta_0 \equiv 1/n\alpha$, the tachyon-photons have positive, zero or negative energy. Emission and absorption processes are exchanged correspondingly.

very high velocity) the fundamental ionic level has a (total) energy *larger* than the excited one.

Finally (and although we do not consider it important) we come back to the question of the group velocity of our tachyon photons. We consider the following variations of formula (2.3):

$$\delta k^y = 0, \qquad \omega\delta\omega = k^x\,\delta k^x \tag{2.14a}$$

$$\delta k^x = 0, \qquad \omega\delta\omega = k^y\,\delta k^y \tag{2.14b}$$

whence, according to Rayleigh's well-known formula for the group velocity $\mathbf{v}$,

$$v^x = k^x/\omega, \qquad v^y = k^y/\omega \tag{2.15}$$

With k^x and k^y real, that is, for ordinary plane wave packets, these are the well-known de Broglie formulas. With k^y imaginary and $k^x > 0$, we find v^y imaginary and $v^x > 1$—a typically 'tachyonic' property. But of course, neither with tachyons proper (if they happen to exist) nor with these tachyon-photons (which should exist), is there ground to believe that a *signal* velocity could be greater than one.

3. *Maxwell–Minkowski Tensor*

Apart from a common multiplicating factor

$$P \equiv \exp[j\omega(t - n\alpha x) - (n^2\alpha^2 - 1)^{1/2}\,\omega y] \tag{3.1}$$

expressing their space-time dependence and containing the complex propagation vector $\mathbf{k}$ of components

$$k^x = n\alpha\omega, \qquad k^y = -j(n^2\alpha^2 - 1)^{1/2}\,\omega, \qquad k^z = 0 \tag{3.2}$$

the components of the electromagnetic field inside Fresnel's evanescent wave are

'Electric wave' contribution:

$$E^z \equiv E^z, \qquad H^x = -j(n^2\alpha^2 - 1)^{1/2} E^z, \qquad H^y = -n\alpha E^z \tag{3.3}$$

'Magnetic wave' contribution:

$$H^z \equiv H^z, \qquad E^x = j(n^2\alpha^2 - 1)^{1/2} H^z, \qquad E^y = n\alpha H^z \tag{3.4}$$

One of course easily verifies that both contributions separately satisfy Maxwell's equations for the vacuum; in fact, the equations in (3.3) and (3.4) are, with reference to (3.1), and after division by $j\omega$, four of the eight Maxwell equations. Thus the most general evanescent wave depends on the three parameters $n\alpha$, E^z and H^z (the latter two being complex).

The components of the Maxwell–Minkoswki energy-momentum tensor M^{ij} inside the evanescent wave are, in the metric x, y, z, it (Ricard, 1970)

$$\begin{aligned} M^{44} &= w = \tfrac{1}{2}P^*P(E^*E + H^*H) \\ M^{\alpha\beta} &= \tfrac{1}{4}P^*P(E^{*\alpha}E^{\beta} + H^{*\alpha}H^{\beta} + \text{c.c.}) - M^{44}\delta^{\alpha\beta} \\ M^{\alpha 4} &= M^{4\alpha} = -iS^{\alpha} = \frac{i}{4}P^*P(E^{*\gamma}H^{\beta} - E^{*\beta}H^{\gamma} + \text{c.c}) \end{aligned} \tag{3.5}$$

We obtain in the present case

$$M^{ij} = \tfrac{1}{2}\exp[-2(n^2\alpha^2 - 1)^{1/2}\omega y] \times$$

$$\times \begin{bmatrix} -M & 0 & -(n^2\alpha^2-1)^{1/2}N & -in\alpha M \\ 0 & 0 & 0 & 0 \\ -(n^2\alpha^2-1)^{1/2}N & 0 & (1-n^2\alpha^2)M & -in\alpha(n^2\alpha^2-1)^{1/2}N \\ -in\alpha M & 0 & -in\alpha(n^2\alpha^2-1)^{1/2}N & n^2\alpha^2 M \end{bmatrix} \tag{3.6}$$

with

$$M \equiv E^{*z}E^z + H^{*z}H^z \tag{3.7}$$

$$N \equiv j(H^{*z}E^z - H^z E^{*z}) \tag{3.8}$$

At this point let us recall that the components of the energy flux $S^{\alpha} \equiv iM^{4\alpha}$ ($\alpha = 1, 2, 3$) have been experimentally tested, iM^{4x} and iM^{4z} yielding, respectively, the correct predictions (Imbert, 1968; Ricard, 1970) for the longitudinal Goos–Hänchen (Goos & Hänchen, 1947) displacement and the new transverse displacement of the reflected beam in total reflection (Imbert, 1969, 1970a, b). In *this* respect the Maxwell–Minkowski tensor is thus quite satisfactory.

In an other respect, however, it is very unsatisfactory, as the expressions $iM^{\alpha 4}$ it gives for the momentum density are not compatible with those of the momentum quanta discussed in Section 2. First, iM^{z4} is non-zero in general and the integral

$$i\int_{-\infty}^{0} M^{z4}\,dy$$

is accordingly non-zero, while the z component of the momentum quanta is identically zero. Second, iM^{x4}/M^{44} has the constant value $1/n\alpha < 1$, while the ratio k^x/ω of the x component of the momentum quanta to the energy quanta is $n\alpha > 1$ (tachyon photons). Thus, the (symmetric) Maxwell–Minkowski energy-momentum tensor cannot be considered as a density appropriately associated with the energy-momentum quanta carried by the evanescent wave. The reason for this is fairly obvious: its connection with the generator of space-time displacements ∂_i is too indirect.†

This, of course, is not to say that the Maxwell–Minkowski tensor does not describe adequately the energy-momentum exchanges between wave and medium on the macroscopic level: it certainly does. What we are saying is that, due to its prequantal origin, the Maxwell–Minkowski tensor does not allow an appropriate discussion of what is going on at the level of spinning photons. In particular, it does not distinguish in the macroscopic momentum density what is due to the photons momentum proper and what is due to the so-called boost [in relation with the $(\alpha,4)$ components of the six-component angular momentum]. To this we intend to come back in a subsequent paper.

For the present we will restrict ourselves to the discussion of the energy-momentum density canonically associated with the energy-momentum quanta inside Fresnel's evanescent wave.

4. *De Broglie's Energy Momentum Tensor*

L. de Broglie‡ proposes as the canonical energy-momentum tensor associated with the generator ∂_i of space-time displacements (that is, with the quantal energy-momentum operator $j\partial_i$), the (asymmetric) tensor

$$T^{ij} = A_K[\partial^i]\, B^{jk} \tag{4.1}$$

$(i,j,k,l = 1,2,3,4;\ x^4 = it)$ or, in the case of complex fields,

$$T^{ij} = \tfrac{1}{8} A_k{}^*[\partial^i]\, B^{jK} + \text{c.c.} \tag{4.2}$$

where $[\partial^i] \equiv \underset{\rightarrow}{\partial^i} - \underset{\leftarrow}{\partial^i}$ is the Gordon operator and

$$B^{ij} \equiv \partial^i A^j - \partial^j A^i, \ \textit{with}\ \partial_i A^i = 0 \tag{4.3}$$

the electromagnetic field derived from 4-potential A^i. In both cases of a true ($k^i \equiv \mathbf{k}$, $i\omega$ real) or inhomogeneous (k^i complex) plane wave, one has

$$k_i k^i = 0 \tag{4.4}$$

and

$$jB^{ij} = k^i A^j - k^j A^i, \qquad k_i A^i = 0 \tag{4.5}$$

† A. J. Janis [*American Journal of Physics*, **38**, 202 (1970)] gives a concise presentation of the connection between the M^{ij} tensor and space-time displacements. See also footnote ‡ on page 127.

‡ See footnote † on page 127.

Then T^{ij} is the product of two 4-vectors

$$T^{ij} = \tfrac{1}{8}(k^i + k^{*i})\,[A^j A_l^* k^l + A^{*j} A_l k^{*l} - A_l^* A^l (k^j + k^{*j})] \qquad (4.6)$$

one, $(^i)$, closely related with the energy-momentum and the other, $(^j)$, with the energy flux or (more loosely speaking) with the 4-velocity of the photons (de Beauregard, 1942, 1943; Weyssenhof, 1947; Weyssenhof & Raabe, 1947).

For both the true or the inhomogeneous plane wave the gauge potential is

$$A^i = ak^i \exp(-jk_l x^l)\,(a \text{ is arbitrary}) \qquad (4.7)$$

For the true plane wave, k^i being real and according to formulas (4.4) and (4.5), the T^{ij} tensor is gauge invariant. But, in the inhomogeneous plane wave, as in general $k_i^* k^i \neq 0$ and $A_i^* k^i \neq 0$, the T^{ij} tensor will not be gauge invariant. To this we will come back later. Presently we will calculate the T_0^{ij} of our evanescent wave in the particular gauge which is transverse in the diopter's rest frame:

$$\mathbf{k}.\mathbf{A}_0 = 0, \qquad A_0{}^4 \equiv iV_0 = 0 \qquad (4.8)$$

so that

$$\mathbf{E} = -j\omega \mathbf{A}_0 \qquad (4.9)$$

We thus obtain

$$T_0^{ij} = \tfrac{1}{2}\exp[-2(n^2\alpha^2 - 1)^{1/2}\omega y] \times$$

$$\times \begin{bmatrix} -n^2\alpha^2 M & 0 & -n^2\alpha^2(n^2\alpha^2-1)^{1/2} N & -in\alpha M' \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -in\alpha M & 0 & -in\alpha(n^2\alpha^2-1)^{1/2} N & M' \end{bmatrix} \qquad (4.10)$$

where M, N and M' are defined by (3.7), (3.8) and

$$M' = E^{*z}E^z + (2n^2\alpha^2 - 1)\,H^{*z}H^z \qquad (4.11)$$

Comparing (3.6) and (4.10) we notice that $(\alpha = 1, 2, 3)$

$$T^{4\alpha} = M^{4\alpha} = M^{\alpha 4} = iS^\alpha \qquad (4.12)$$

thus, the energy flux is the same in the Maxwell–Minkowski tensor and in the de Broglie tensor T_0^{ij}; that is, the de Broglie tensor predicts just as correctly as the Maxwell–Minkowski tensor the longitudinal Goos–Hänchen (Goos & Hänchen, 1947) and the recent (Imbert, 1969, 1970a, b) transverse displacements.

On the other hand, by its very definition the de Broglie (asymmetric) tensor is perfectly compatible with the concept of the energy-momentum quanta of the wave. In our case, as $k^z = 0$, T^{zi}, and in particular $T^{z4} = 0$: there are neither momentum quanta nor momentum density in the z direction. The unsatisfactory fact that $M^{z4} \neq 0$ with the Maxwell–Minkowski tensor is thus avoided. Similarly $iT^{x4}/T^{44} = n\alpha > 1$ closely corresponds to the 'tachyon' property of energy-momentum quanta $k^x/\omega = n\alpha > 1$, and contrasts the ratio $iM^{x4}/M^{44} = 1/n\alpha < 1$ obtaining with Maxwell's tensor.

We thus conclude that the asymmetric de Broglie tensor, expressed in the gauge which is transverse in the diopter's rest frame, is (contrary to the Maxwell–Minkowski tensor) completely satisfactory in both its descriptions of the energy flux and of the momentum density.

Now, as we have seen, in the case of an inhomogeneous plane wave the de Broglie tensor (4.6) is not gauge invariant. We must thus examine the gauge dependent T_1^{ij} which is then added to the preceding T_0^{ij}, the expression of which is, according to (4.7),

$$T_1^{ij} = \tfrac{1}{8}(k^i + k^{*i})(a^* k_l^* k^l A_0{}^j - a^* A_0{}^l k_l^* k^j \exp(-jk_l x^l) + \text{c.c.}) \quad (4.13)$$

According to (3.2), (4.9) and (3.4)

$$k_l^* k^l = 2(n^2\alpha^2 - 1)\,\omega^2$$

$$A_0{}^l k_l^* = -2n\alpha(n^2\alpha^2 - 1)^{1/2} H^z \exp(-jk_1 x^1)$$

so that, setting

$$R \equiv \omega(a^* H^z + aH^{*z}), \qquad S \equiv j\omega(a^* \dot{E}^z - aE^{*z}) \quad (4.14)$$

$$in\,\alpha\tfrac{1}{2}(^j) = [(n^2\alpha^2 - 1)^{1/2} R,\ 0,\ (n^2\alpha^2 - 1)\,S,\ in^{\alpha}(n^2\alpha^2 - 1)^{1/2} R] \times$$
$$\times \exp[-2(n^2\alpha^2 - 1)^{1/2}\,\omega y] \quad (4.15)$$

and thus

$$T_1^{ij} = \tfrac{1}{2}\exp[-2(n^2\alpha^2 - 1)^{1/2}\,\omega y] \times$$

$$\times \begin{bmatrix} n\alpha(n^2\alpha^2 - 1)^{1/2} R & 0 & n\alpha(n^2\alpha^2 - 1)\,S & in^2\alpha^2(n^2\alpha^2 - 1)^{1/2} R \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i(n^2\alpha^2 - 1)^{1/2} R & 0 & i(n^2\alpha^2 - 1)\,S & -n\alpha(n^2\alpha^2 - 1)^{1/2} R \end{bmatrix} \quad (4.16)$$

Thus there is in (4.16), through the definitions (4.14), an additive contribution, proportional to the arbitrary constant a, to all the terms of (4.10). But, as the values of the energy fluxes in the x and z directions have been measured through the longitudinal and transverse shifts they impart to the reflected beam and have been found equal to their calculated values, $-iT^{4x}$ and $-iT^{4z}$, we must have (Imbert, 1968, 1969, 1970a, b; Ricard, 1970) (discarding the limiting case of total reflection $n^2\alpha^2 - 1 = 0$) $R = S = 0$, that is, according to (4.4),

$$(E^{*z} H^z + E^z H^{*z})\,a = 0 \quad (4.17)$$

Thus (except in the four cases where the parenthesis is zero: $E^z = 0$, $H^z = 0$, $H^z = \pm iE^z$, that is, linear polarization parallel or perpendicular to the incidence plane, left or right circular polarization inside the evanescent wave), we conclude $a = 0$. In other words, as discrete exceptions do not invalidate a general conclusion, the gauge must be the transverse gauge in the rest frame of the medium.

5. *Conclusions*

As a direct consequence of Fresnel's theory of the evanescent wave associated with total reflection, the light quanta in it have (in units such that $c = 1$ and $\hbar = 1$) an imaginary component of their momentum in the direction orthogonal to the reflecting plane, and, correspondingly, a projection k^x of their momentum on the reflecting plane that is larger than their angular frequency ω. We have shown that a typical tachyon property should follow from this, namely, the possibility of simultaneous absorption (or stimulated emission) of the energy quantum ω and the momentum quantum $k^x > \omega$.

Then we have shown that the classical Maxwell–Minkowski energy-momentum tensor M^{ij} does not provide an adequate description of the properties of the light quanta inside the evanescent wave (which we assume for simplicity to be *in vacuo*). First, as expressed in the metric $i,j,k,l = x,y,z,it$, the (constant) ratio $iM^{x4}/M^{44} \equiv S^x/w$ of the x component of the Poynting vector to the energy density is $1/n\alpha < 1$, whereas, as said above, $k^x/\omega = n\alpha > 1$ (n denotes the index of the medium and α the incidence angle's sine in the incident plane wave). Thus, as far as the description of light quanta inside the evanescent wave is at stake, the Maxwell–Minkowski tensor does not give the right value for the ratio of momentum density in the x direction to energy density.

It does not give either the right value of the momentum density in the direction z orthogonal to the incidence plane. In all cases except those of linear polarization parallel or perpendicular to the incidence plane, $iM^{z4} \equiv S^z$ is non-zero, with the same spatial dependence

$$\exp[-\tfrac{1}{2}(n^2\alpha^2 - 1)^{1/2}\omega y]$$

as all other components of M^{ij}. Thus the integrated value $\int_0^\infty S^z dy$ is not zero, while of course the k^z component of the photons momentum is identically zero. So we find again an incompatibility between the concept of energy-momentum quanta inside the evanescent wave and the expression of the momentum density provided by the Maxwell–Minkowski tensor.

However, if the components iM^{x4} and iM^{z4} of the momentum density are, as we have just said, unsatisfactory, the components iM^{4x} and iM^{4z} of the energy flux are quite satisfactory, as appropriate experiments have tested their values within the range of experimental accuracy. We are here referring to Kristoffel's (1956) and Renard's (1964) theory of the longitudinal Goos-Hänchen (Goos & Hänchen, 1947) shift and to Imbert's (1968) theory of the new transverse shift (Imbert, 1969, 1970a, b) of the reflected beam in total reflection, both of which are similar and make use of the energy fluxes

$$\int_0^\infty dy \int_{Z_1}^{Z_2} dz\, S^x \qquad \text{and} \qquad \int_0^\infty dy \int_{X_1}^{X_2} dx\, S^z$$

Considering in particular the components $(z,4)$ and $(4,z)$ of the energy-momentum tensor, it is then clear that the only way to reconcile the zero value the first one should have with the non-zero value the second one rightly has in general, is to use an energy-momentum tensor which is asymmetric (even in the vacuum). This, of course, excludes the Maxwell–Minkowski tensor, which is essentially symmetric in the vacuum.

The reason why the Maxwell–Minkowski tensor is not appropriate as an energy-momentum density associated with photons propagating in the vacuum is fairly obvious: it is not canonically related to the generator ∂_i of space-time translations.† Now, there *is* a tensor representing the energy-momentum density canonically related to the operator ∂_i: de Broglie's‡ tensor T^{ij} of expression (4.2). It is, moreover, an asymmetric tensor (even in the vacuum), and it is such by virtue of its close relation not only to the energy-momentum operator $j\partial_i$, but also to the spin properties of the photon. This brings us back to early considerations by one of us (de Beauregard, 1942, 1943) and by Weyssenhof (1947) on the non-collinearity of velocity and momentum for spinning particles and spinning media. *This* was indeed the driving idea (de Beauregard, 1965) leading to the recent calculations (Imbert, 1968) and experiments (Imbert, 1969, 1970a, b) on the transverse shift by total reflection of a circularly polarized light beam.

At this point it is significant to remark that (*modulo* a multiplicative constant) de Broglie's tensor $A_k(\partial^i)B^{jk}$ is the *only* second rank tensor, quadratic in the electromagnetic field magnitudes, and containing the Gordon operator $[\partial^i] \equiv \underset{\rightarrow}{\partial^i} - \underset{\leftarrow}{\partial^i}$ with the index i free. Now, to say that the energy-momentum density associated with propagating photons should be described by de Broglie's tensor (4.1) or (4.2) is to confer *ipso facto* some physical reality to the electromagnetic potential A^i. The natural question is then, can we find situations where the de Broglie tensor T^{ij} is not gauge invariant *and* where some of its components are physically measurable? The answer is twice *yes*—in Fresnel's evanescent wave.

We have shown in Section 3 that, when expressed in the gauge that is transverse in the medium's rest frame, de Broglie's tensor yields the values $iT^{4x} = iM^{4x} = S^x$ and $iT^{4z} = iM^{4z} = S^z$ that have been proved experimentally to be the good ones (by measurements of the longitudinal and transverse shifts of the reflected beam). But, contrary to the Maxwell–Minkowski tensor, the asymmetric de Broglie tensor also yields the good value $iT^{z4} = 0$ and the good ratio $iT^{x4}/T^{44} = k^x/\omega = n\alpha > 1$—this following from its direct relation to the generator ∂_i of space-time translations. Thus, as expressed in the local transverse gauge, the de Broglie energy-momentum tensor is completely satisfactory in both respects of the (measured) energy fluxes and of the momentum densities.

Then we have shown that the additional term arising in T^{ij} from the

† A. J. Janis [*American Journal of Physics*, **38**, 202 (1970)], gives a concise presentation of the connection between the M^{ij} tensor and space-time displacements.

‡ See footnote † on page 127.

arbitrary gauge dependence would spoil everything, as far as the energy fluxes are at stake. Whence a necessary conclusion: the measured values of the energy fluxes iT^{4x} and iT^{4z} unequivocally select one gauge: the one that is transverse in the medium's rest frame.

This conclusion is quite similar to (but drawn from experiment in a much more compelling way than) an earlier conclusion by de Broglie (1949a) following an analysis of angular momentum in expanding spherical waves. Our conclusion inclines us to believe that this selection of transverse potentials, as being those on which the medium exerts an action, has a natural corollary: that longitudinal potentials could also exist but with an extremely weak interaction with matter. In other words, our work leads us to give weight to be Broglie's theory that the photon has an exceedingly small but non zero rest mass, and that spin-0 photons thus exist, but with an extremely weak interaction with matter (so that, among other things, the black-body radiation theory has to deal with two, and not three, spin states of the photon) (de Broglie, 1949b; Bass & Schrödinger, 1955).

We have recently taken cognizance of papers where the existence of a transverse energy flux inside an inhomogeneous valve wave is discussed (without any calculation of the corresponding external shift):

Boguslawski, S. (1912). *Physikalische Zeitschrift*, **13**, 393.
Wiegrefe, A. (1914). *Annalen der Physik*, **44**, 887.
Fedorov, F. I. (1955). *Doklady Akademi i nauk SSSR*, **105**, 465.

The difference between the energy fluxes and the momentum densities is not discussed in these papers.

Caruiglia, C. K. and Mandel, S. (1971), *Physical Review*, **3D**, 880, have treated the quantisation of electromagnetic evanescent waves. The commutation relations they obtain in the k-representation [equations (66), (67), (82) and (83)] are quite consistent with our 'tachyon photons'.

Glass, S. J. and Mendlowitz, H. (1968), *Physical Review*, **174**, 57, explain the Smith–Purcell experiment in terms of a first-order photoelectric effect occurring near a metallic grating.

References

Aharonov, Y. and Bohm, D. (1959). *Physical Review*, **115**, 485.
Bass, J. and Schrödinger, E. (1955). *Proceedings of the Royal Society*, **A232**, 1.
de Beauregard, O. Costa (1942). *Compte rendu hebdomadaire des séances de l'Académie des sciences*, **214**, 904.
de Beauregard, O. Costa (1943). *Journal de mathématiques pures et appliquées*, **22**, 85 (see especially pp. 128–36).
de Beauregard, O. Costa (1965). *Physical Review*, **139**, B1443. See also, *Perspectives in General Relativity* (ed. B. Hoffmann), p. 44. Indiana University Press.
de Beauregard, O. Costa (1970). *Compte rendu hebdomadaire des séances de l'Académie des sciences*, **270**, B773 and B1004.
Boersch, H. (1962). *Zeitschrift für Physik*, **167**, 72.
Brillouin, L. (1964). *Proceedings of the National Academy of Sciences of the United States of America*, **53**, 475 and 1280.

de Broglie, L. (1947). *Compte rendu hebdomadaire des séances de l'Académie des sciences*, **255**, 163.
de Broglie, L. (1949a). *Mécanique Ondulatoire du Photon et Théorie Quantique des Champs*, pp. 72–8. Gauthier-Villars, Paris.
de Broglie, L. (1949b). *Mécanique Ondulatoire du Photon et Théorie Quantique des Champs*, pp. 62–4. Gauthier-Villars, Paris.
de Broglie, L. (1950). *Optique Electronique et Corpusculaire*, pp. 45–9. Herrmann, Paris.
Chambers, R. G. (1960). *Physical Review Letters*, **5**, 3.
Ehrenberg, W. and Siday, R. E. (1949). *Proceedings of the Physical Society*, **B62**, 21.
Frenkel, J. (1926). *Zeitschrift für Physik.*, **37**, 243.
Goos, F. and Hänchen, H. (1947). *Annals of Physics*, **1**, 133.
Imbert, Ch. (1968). *Compte rendu hebdomadaire des séances de l'Académie des sciences*, **267**, B1401.
Imbert, Ch. (1969). *Compte rendu hebdomadaire des séances de l'Académie des sciences*, **269**, B1227.
Imbert, Ch. (1970a). *Compte rendu hebdomadaire des séances de l'Académie des sciences*, **270**, B529.
Imbert, Ch. (1970b). *Physical Review Letters*, **31A**, 337.
Imbert, Ch. and Ricard, J. (1968). *Compte rendu hebdomadaire des séances de l'Académie des sciences*, **266**, B131.
Imbert, Ch. and Ricard, J. (1970). *Compte rendu hebdomadaire des séances de l'Académie des sciences*, **270**, B1096.
Kristoffel, N. (1956). *Scient. Acta Tartu Univ.*, private communication.
Mathisson, M. (1931). *Zeitschrift für Physik*, **67**, 270 and 826.
Mathisson, M. (1933). *Mathematische Annalen*, **107**, 400.
Mathisson, M. (1937). *Acta physica polonica*, **6**, 163 and 218.
Proca, A. (1933). *Annales de physique*, **20**, 347.
Renard, R. H. (1964). *Journal of the Optical Society of America*, **54**, 1190.
Ricard, J. (1970). *Compte rendu hebdomadaire des séances de l'Académie des sciences*, **270**, B381 and 661.
Weyssenhof, J. (1947). *Acta physica polonica*, **9**, 26, 34 and 46.
Weyssenhof, J. and Raabe, A. (1947). *Acta physica polonica*, **9**, 7 and 19.