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Noncollinearity of Velocity and Momentum of Spinning Particles

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A theoretical and experimental search for the so-called Weyssenhof behavior of a spinning particle, due to the noncollinearity of its velocity and momentum, has been undertaken. Z-independent solutions of Maxwell's equations had previously been produced with a nonzero s^z *component of the Poynting vector; indeed, Imbert emphasized that the spatial exponential damping of Fresnel's evanescent wave would entail a nonzero value for the integral* $\iint s^z \, dx \, dy$. *Excellent experimental verifications of this point have been obtained by Imbert. Besides having no* z *component of their momentum, the energy-momentum quanta inside Fresnel's evanescent wave have typical tachyon properties, the imaginary character of their* y *component (normal to the reflecting surface) entailing that (in units such that* $c = 1$*) their* x *component is* larger *than the energy quanta. Imbert is now planning experiments to test these interesting properties. Thus, the two main aspects of noncollinearity of velocity and momentum of spinning particles would be displayed.*

1. INTRODUCTION

Two related ideas (although, strictly speaking, not connected reciprocally to each other) have been proposed again and again by a few theoretical physicists. The first idea is that the energy-momentum tensor T^{ij} ($i, j, k, l = 1, 2, 3, 4$; $x^4 = ict$) appropriate for media endowed with a spin density $\sigma^{[ij]k}$ should be asymmetric and obey (in flat space–time) the formula

$$T^{ij} - T^{ji} = \partial_k \sigma^{ijk} \tag{1}$$

∂_k being the divergence operator. It should be remembered that the classical theory of stressed continuous media supporting not only a force density $f^\alpha(\alpha, \beta, \gamma = 1, 2, 3)$, but also a torque density $f^{[\alpha\beta]}$, uses an asymmetric elastic tensor $E^{\alpha\beta}$ obeying the formula

$$E^{\alpha\beta} - E^{\beta\alpha} = \mu^{\alpha\beta} \tag{2}$$

which, when transposed in relativistic form and compared with (1), yields the very appropriate formula

$$\partial_k \sigma^{ijk} = \mu^{ij} \tag{3}$$

between spin and torque densities. Incidentally, the condition for equality of the two divergences

$$\partial_j T^{ij} = \partial_j T^{ji} = f^i \tag{4}$$

is complete skew-symmetry of the spin density σ^{ijk}; the close analogy between formulas (3) and (4) should be noted.

The derivation of formulas (1) and (3) is quite analogous to the classical derivation of formula (2); it has been given quite often and need not be repeated here. It is also part of the derivation of the well-known Noether formulas. Also, formula (1) arises in Dirac's theory for the spinning electron, σ then being Dirac's completely skew-symmetric $\sigma^{[ijk]}$, and T^{ij} Tetrode's[1] canonical asymmetric energy-momentum tensor. It arises also quite naturally in almost all theories of spinning particles, for instance, in the Petiau–Duffin–Kemmer theory of spin -1 particles or, using de Broglie's[2] formulas for the photon's spin density, in both the Maxwell and the de Broglie–Proca theories of the photon.

Among the authors who have used this kind of approach to the theory of spinning media, we cite ourselves,[3] Weyssenhof and Raabe,[4] Papapetrou,[5] Sciama,[6] and Kibble,[7] the latter three being interested in gravitational problems and non-Minkowskian metrics and connections.

The second idea, which is manifestly related to the preceding one, although not in a one-to-one fashion because of obvious boundary value problems, is that the space–time 4-velocity of a spinning point particle is noncollinear with its momentum. Here again there exists a classical analogy:

the so-called Weyssenhof dynamical equations of a spinning point particle,

$$F^i = p'^i_\tau \tag{5a}$$

$$M^{ij} = V^i p^j - V^j p^i + S'^{ij}_\tau \tag{5b}$$

are isomorphic[8] to the classical equations of the statics of a stiff filament (resisting curvature and torsion):

$$\mathbf{f} = \mathbf{T}_s' \tag{6a}$$

$$\eta = \mathbf{t} \times \mathbf{T} \times \gamma_s' \tag{6b}$$

The isomorphism is as shown in Table I. This isomorphism also holds in the derivation of the "first" and "second" equations (the latter vanishing if the filament is perfectly flexible, or if the point particle is spinless). A characteristic feature of the "second equation" is the presence of the cross products $\mathbf{t} \times \mathbf{T}$ or $V^i p^j - V^j p^i$, entailing noncollinearity of the tension $\mathbf{T}$ and the unit vector $\mathbf{t}$ tangent to the filament, or of the momentum–energy p^i and the 4-velocity V^i. Thus, as is very well known, the figure of a stiff filament festoons around the one of a flexible filament subjected to identical external tensions; however, it cannot depart far from the figure of the flexible filament. Similarly, the space–time trajectory of a spinning point particle will slightly

Table I

Classical statics of a stiff filament (Three-dimensional Euclidean space)		Relativistic dynamics of a spinning point particle (Four-dimensional Minkowskian space–time; $x^4 = ict$; $i, j, k, l = 1, 2, 3, 4$)	
Curvilinear abscissa	s	Proper time	τ
Point on the filament	$\mathbf{r}$	Point-instant	x^i
Tangent unit vector	$\mathbf{t} \equiv \mathbf{r}_s'$	4-velocity ($V_i V^i = -c^2$)	$V^i \equiv x'^i_\tau$
Tension	$\mathbf{T}(s)$	Momentum-energy	$p^i(\tau)$
Linear density of applied force	$\mathbf{f}(s)$	Applied 4-force	$F^i(\tau)$
First equation [(6a)]	$\mathbf{f} = \mathbf{T}_s'$	First equation [(5a)]	$F^i = p'^i_\tau$
Angular tension (resisting torque)	$\gamma(s)$	Internal angular momentum (spin)	$s^{[ij]}(\tau)$
Linear density of applied torque	$\eta(s)$	Applied torque	$M^{[ij]}(\tau)$
Second equation [(6b)]	$\eta = \mathbf{t} \times \mathbf{T} + \gamma_s'$	Second equation [(5b)]	$M^{ij} = V^i p^j - V^j p^i + S'^{ij}_\tau$

festoon around the trajectory of a spinless point particle embedded in the same space–time field. The same will be true in ordinary space if the external field is static, that is, time-independent in a particular Lorentz frame. This is known as the "Weyssenhof behavior" of a spinning particle, and "corresponds" classically to Schrödinger's *Zitterbewegung* of Dirac's electron.

As for the magnitude of the wandering of the spinning particle, it turns out to be of the order of the wavelength of the associated matter wave; the basic reason for this is that Planck's constant h is the quantum of both ($4n\pi$ times) the angular momentum and of the action.

Of course, anyone taking the considerations we have noted seriously should be eager to devise an experiment displaying the Weyssenhof wandering of a spinning particle. There are two difficulties in this. First, the experimental procedure must be such that the associated boundary value problem will not destroy the phenomenon, as will be the case in all unsophisticated arrangements, due to the "integral equivalence" of the two expressions $\iiint T^{ij}\, du_j$ and $\iiint T^{ji} du_j$ whenever the contour integral at infinity $\iint \sigma^{ijk} ds_{jk}$ is zero; by definition, $6icdu_i \equiv \epsilon_{ijkl}[dx^j\, dx^k\, dx^l]$ and $2ic\, ds_{ij} \equiv \epsilon_{ijkl}[dx^k\, dx^l]$; the integrals are taken over a three-dimensional surface extending to infinity.

If this difficulty is overcome, a second one will appear: the expected spatial displacement being of the order of the wavelength, it will lie, so to speak, almost "inside" the diffraction pattern of any detecting apparatus, so displaying it will require much experimental ingenuity and skill. Our colleague, the experimentalist Dr. Christian Imbert (who is also an adept theorist in optics) has superbly solved these two difficulties in turn, as will be shown in the following. As the calculations[9] and the measurements[10] of his experiments have been published, we will refer the reader to them for more details.

The first and brilliant suggestion by Imbert was that one would get rid of the "integral equivalence" difficulty by choosing an arrangement where a spatial exponential damping exists; absorbing media, however, should be discarded, because energy absorption will obviously render observation practically impossible. There is in optics, however, an interesting, long known phenomenon where an exponential damping factor is present and entails no energy absorption: Fresnel's evanescent wave associated with total reflection. This is indeed the first known tunnel effect, if by tunnel effect we mean a problem where conservation of energy and/or momentum is possible only with complex values of these quantities. As is well known, Fresnel's evanescent wave is formally a plane wave with a complex normal unit vector, the two components of which normal and parallel to the reflecting plane are respectively imaginary, and real but larger than one. In other words, and in units such that $c = 1$, the corresponding components of the propagation vector are imaginary, and real, but larger than the angular frequency ω;

or the corresponding components of the photon's momentum are imaginary, and real, but larger than the energy. Thus, the photons tunneling inside Fresnel's evanescent wave have very peculiar properties, in that not only do they exhibit a Weyssenhof behavior, but they also display tachyon properties (the imaginary component of their momentum entailing consequences similar to those of the imaginary proper mass of tachyons proper). Thus, Fresnel's almost infallible physical intuition and theoretical genius made him hit here upon something extremely significant even today.

Apart from the fundamental advantage of working with Fresnel's evanescent wave, the use of spinning photons in the domain of classical optics is interesting because, although the effective length unit is of course the wavelength itself, the fact that this wavelength is of the order of 4000–8000 Å will make measurements more convenient than would be the case with, say, electrons or protons. For the other features of Imbert's very ingenious experiments, we refer the reader to his publications, and will now proceed with the theory of what we have termed the "photon's translational inertial spin effect."

2. A PURPOSELY SELECTED SOLUTION OF THE VACUUM MAXWELL EQUATIONS

The property of spinning particles that we intend to bring to an experimental test is of course a quite general one, which can be deduced from the equations of any spinning particle, as, for instance, Dirac's electron.[11]. The reason we have selected the case of the photon is not only that the wavelength range is convenient to work with, but also that the techniques of either classical optics or high-frequency radio waves make it quite easy to reflect, refract, or polarize the photons. So we will, first of all, select *ad hoc* a class of vacuum solutions of Maxwell's equations.

Let ψ denote any of the ten components of the vacuum photon wave (**E**, **H**, **A**, V) in rectangular Cartesian coordinates x, y, z, under the special assumption

$$\partial_z \psi \equiv 0 \tag{7}$$

so that *the photon's momentum has no z component.* For simplicity, we assume a "harmonic" time dependence; that is, the time dependence of the ten ψ components will be through a common factor $\exp(i\omega t)$:

$$\psi \equiv \psi_0 \exp(i\omega t) \tag{8}$$

Then, ψ_0 is a solution of the so-called Helmholtz equation

$$(\partial_x^2 + \partial_y^2 + \omega^2)\, \psi_0 = 0 \tag{9}$$

Now, we superpose (with the same ω) a so-called "electric-type solution" (E)[1]

$$E_z \equiv E_z\,, \qquad H_x = +j\omega^{-1}\,\partial_y E_z\,, \qquad H_y = -j\omega^{-1}\,\partial_x E_z \tag{10}$$

and a "magnetic-type solution" (H)

$$H_z \equiv H_z\,, \qquad E_x = -j\omega^{-1}\,\partial_y H_z\,, \qquad E_y = +j\omega^{-1}\,\partial_x H_z \tag{11}$$

(all the other components being zero).

In (8), we have taken the photon wave function ψ to be complex: as Dirac(12) and de Broglie(2) have both pointed out, this should be taken as a fundamental character of the (nonsecond-quantized) photon wave function—and a property playing an important role in our subsequent deduction.[2]

From the formulas (10) and (11), and the definition of the Poynting vector

$$\mathbf{s} \equiv \tfrac{1}{4}(\mathbf{E}^* \times \mathbf{H} + \mathbf{E} \times \mathbf{H}^*) \tag{12}$$

the following expressions are obtained:

$$s_{x,y} = (j/4\omega)[E_z{}^*(\partial_{x,y})\,E_z + H_z{}^*(\partial_{x,y})\,H_z] \tag{13}$$

$$s_z = (1/4\omega^2)[\partial_x E_z{}^*\,\partial_y H_z - \partial_x H_z{}^*\,\partial_y E_z) + \text{c.c.} \tag{14}$$

where

$$[\partial] \equiv \underset{\rightarrow}{\partial} - \underset{\leftarrow}{\partial} \tag{15}$$

denotes the well-known Gordon current operator. While there is nothing surprising connected with the x and y components (13) of the Poynting vector, the existence of the nonzero s_z component, according to (14) [and in contrast with (7)] deserves more attention. First and very significantly, it depends on the relative phase of the electric (E) and magnetic (H) solutions.

From (13), we easily calculate

$$\partial_x s_y - \partial_y s_x = (j/2\omega)(\partial_x E_z{}^*\,\partial_y E_z + \partial_x H_z{}^*\,\partial_y H_z) + \text{c.c.} \tag{16}$$

[1] We must of course distinguish the j ($j^2 = -1$) arising from the optical or quantal formalisms from the i ($i^2 = -1$) arising from the Space–time metric.

[2] This point needs commentary. In an interesting article, Ricard(13) has given a very understandable classical explanation of both the longitudinal Goos–Hänchen shift and the new transverse Imbert shift by using real solutions of Maxwell's equations, with sines and cosines replacing the e's with imaginary exponents. The point is, however, that many of the significant and elegant formulas implying the complex ψ's and ψ^*'s are lost in such a formalism.

Now, if the boundary conditions are such as to allow that, a denoting a real constant,

$$H_z(x, y) = jaE_z(x, y) \tag{17}$$

then (16), (17), and (14) entail

$$s_z = [1/\omega(a + a^{-1})](\partial_x s_y - \partial_y s_x) \tag{18}$$

This formula is closely connected with the fundamental formula (1), as can be shown[14] by inspection of de Broglie's[2] formulas for the photon's canonical momentum-energy tensor, and Dirac-like current and spin densities.

Formula (18) assumes the integral form

$$\iint s_z \, dx \, dy = [1/\omega(a + a^{-1})] \oint (s_x \, dx + s_y \, dy) \tag{19}$$

It is thus clear that with almost all usual solutions of Maxwell's equations the transverse energy flux associated with s_z will be zero when integrated over the whole x, y plane.

At this point of our research, Imbert adduced a very clever suggestion that some previous calculations of his[15] had prepared. Why not work with Fresnel's evanescent wave associated with total reflection? This wave is excited in the vacuum, and thus nothing will be lost of the intrinsic character of the photon property we are looking for. The boundary conditions on the reflecting surface are such that the condition (17) is easily satisfied: one has simply to reflect a plane, elliptically polarized wave. And—last but not least—the presence of a real exponential damping factor in the direction y orthogonal to the reflecting surface will entail that the integral (19) assumes a finite value Φ_z when extended over the semiplane $z = 0$, $y < 0$ inside the evanescent wave (or equivalently, to the whole $z = 0$ plane[16]).

Now, we may remember that Kristoffel[17] and Renard[18] have produced a theory of the well-known longitudinal Goos–Hänchen[20] shift in total reflection based upon the existence of the longitudinal energy flux

$$\iint_{z=-\infty}^{z=0} s_x \, dy \, dz$$

inside Fresnel's evanescent wave. Imbert[9] then used a quite analogous calculation to predict a lateral shift by total reflection of an elliptically polarized plane wave, the maximum value of which occurs, as shown by Ricard,[20] when the evanescent wave is (left or right) circularly polarized in the sense that $a = \pm 1$; this does not exactly correspond to circular polarization of either the incident or the reflected plane wave.

However, for an experimental observation of the calculated lateral shift, it is not advisable to use this theoretically optimum case. As the calculated transverse shift is very small, of the order of half the wavelength, that is, some 20 times smaller than the longitudinal shift observed by Goos and Hänchen, it will be necessary to multiply it by N successive reflections. As is well known, in the limiting case of total reflection, the theoretical phase difference between the two linear polarization states parallel and perpendicular to the incidence plane is preserved exactly, and this of course remains almost true in quasi-limit total reflection. Thus, in particular, a pure helicity state of the incident photons will be preserved. This is a very favorable circumstance, that will allow a multiplication by one of the two Imbert devices: (1) an isosceles triangular prism, where total reflection occurs on the "basis" and the light rays hit orthogonally the semireflecting "sides"; each of the latter reflections reverses the helicity sign, which is the condition for adding the transverse shifts (Fig. 1). (2) An equilateral triangular prism,[3] inside of which the light follows a helical regular polygon, the slope of which is ajusted so that successive quasilimit total reflections occur (Fig. 2).

With both of these multiplying devices, and marking the beam with a rectilinear object,[4] the two halves of which were illuminated by left and right circularly polarized light (with the possibility of exchanging the two helicity states by turning a quarter-wave plate) (Fig. 3), Imbert has shown that the

[3] This solution to the experimental problem had been suggested by Goillot.

[4] The rectilinear object is in fact a sophisticated one: the so-called Wolter object,[21] the image of which is almost a δ-function.

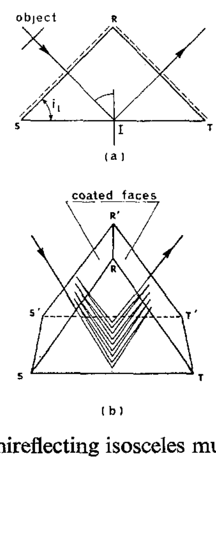

Fig. 1. The semireflecting isosceles multiplying prism.

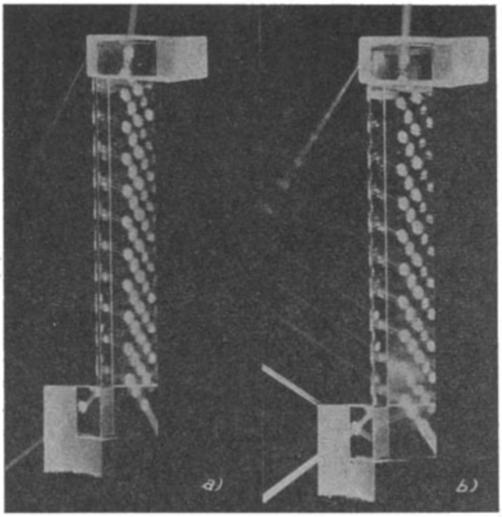

Fig. 2. The equilateral multiplying prism. (a) In working condition (28 total reflections). (b) In a slightly disturbed position showing light escaping.

new calculated transverse shift does indeed exist, with the right sign and magnitude (Fig. 4). In other words, just as the longitudinal Goos–Hänchen shift proves the existence of a longitudinal energy flux in the vacuum of Fresnel's evanescent wave, so the new lateral Imbert shift proves the existence of a corresponding transverse flux when the incident and emergent plane waves are circularly polarized. *In other words, the energy flux of the spinning photons is not collinear with their momentum inside Fresnel's evanescent wave.*

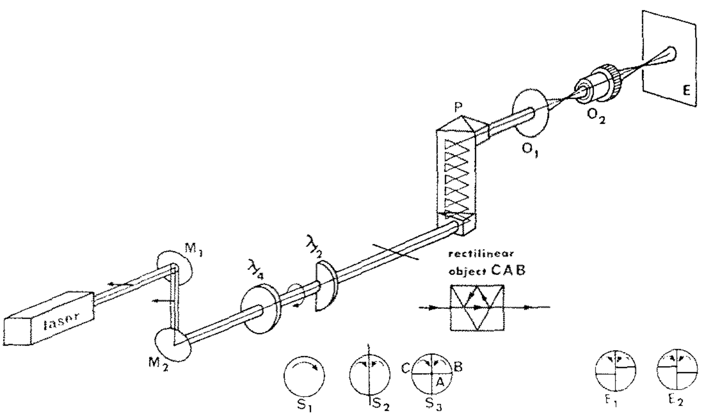

Fig. 3. The overall experimental arrangement.

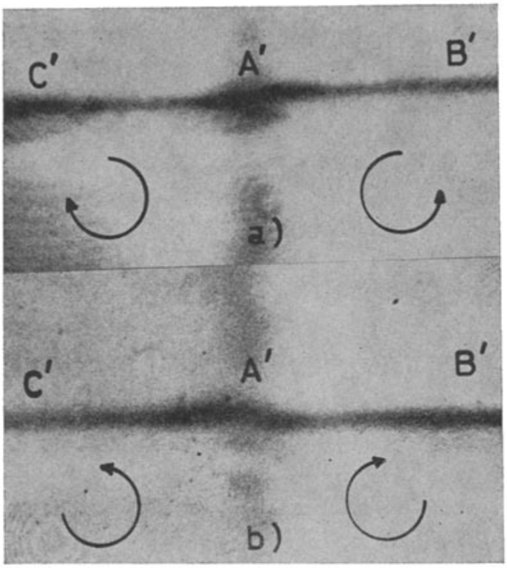

Fig. 4

Figure 5 explains the phenomenon in terms of geometrical optics and using the isomorphism between the statics of stiff filaments and the dynamics of spinning photons explained in Section 10. Suppose a stiff filament is suspended by its extremities and loaded by two weights p_1 as shown in the figure, and that one twists the filament. It is well known (and easily verified) that the curved portion of the filament assumes a helical shape, the helicity sign being the same as that of the twist of the filament.

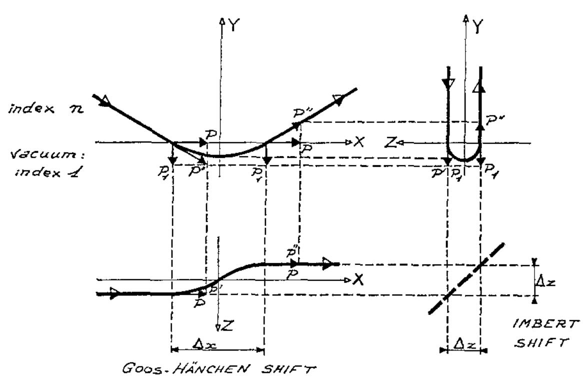

Fig. 5. Total reflection of a spinning photon (positive helicity).

Then, using the same figure, we imagine that all points $y \geqslant 0$ are immersed in a refracting medium of index n and all points $y < 0$ are in the vacuum. The figure may then be taken as displaying the total reflection of a spinning photon, Δx being the longitudinal Goos–Hänchen shift and Δz the new lateral Imbert shift.

3. ENERGY-MOMENTUM QUANTA INSIDE FRESNEL'S EVANESCENT WAVE: TACHYON PHOTONS

It is obviously desirable to experimentally test the properties of the energy–momentum quanta inside Fresnel's evanescent wave, so that the "noncollinearity of velocity and momentum" of the spinning photons is operationally proved in both of its aspects. To this end, we now pursue the theoretical investigation concerning these quanta. For simplicity, we use units such that $c = 1$ and $\hbar = 1$.

Fresnel's evanescent wave is formally a plane wave with a complex propagation vector. Denoting by $\mathbf{k}_i$ the propagation vector of the plane wave incident in the medium of index n

$$(k_i = n\omega > \omega;\ k_i^x = n\alpha\omega > \omega,\ k_i^y = n\beta\omega > 0,\ k_i^z = 0),$$

the complex propagation vector of the evanescent wave has components $k^x = k_i^x = n\alpha\omega > \omega$, $k^y = -j(n^2\alpha^2 - 1)^{1/2}\,\omega$, $k^z = 0$, so that

$$(k^x)^2 + (k^y)^2 = \omega^2 \tag{20}$$

There have already been experiments displaying the absorption of the energy quanta ω inside Fresnel's evanescent wave. We are interested now in simultaneous detection of the absorption (or stimulated emission) of both the energy quanta ω and (complex) momentum quanta $\mathbf{k}$.

At this point, we consider first the transition between two electron states ψ^a and ψ^b. Inserting the preceding energy-momentum quanta in Feynman's expression ($\lambda, \mu = 1, 2, 3, 4$)

$$\mathscr{F} = A_\lambda \iiiint \exp(jk_\mu x^\mu)\,\bar{\psi}^a\gamma^\lambda\psi^b\, d^4x \tag{21}$$

for the transition amplitude, with $x^1 = x$, $x^2 = y$, $x^3 = z$, $x^4 = it$, we see that the imaginary k^y will contribute to the transition amplitude of a "tachyon photon" of momentum $k^x = n\alpha\omega > \omega$, $k^y = k^z = 0$. In other words, if the electron wave function ψ has little extension along the y coordinate, it will

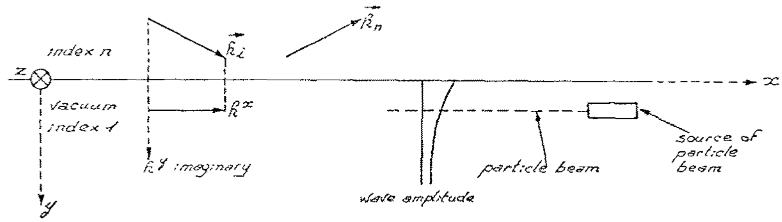

Fig. 6. Beam of particles undergoing transitions inside Fresnel's evanescent wave.

practically feel a rather peculiar "quasi plane wave" of tachyon photons with an exponential y dependence.[5]

This would allow in principle the possibility of a first-order photoelectric effect on free electrons.[23] If (Fig. 6) an electron beam is sent inside the evanescent wave parallel to the x axis with exactly the right velocity β_E (or β_F), it would absorb (or emit) a tachyon photon of energy ω and momentum $k^x = n\alpha\omega > \omega$ (Fig. 7). However, this (very striking) thought-experiment is not a feasible one, because a large value of the energy quanta ω and of the penetration depth of the wave $1/(n^2\alpha^2 - 1)^{1/2}\,\omega$ are contradictory requirements. Before we proceed, let it be clear that the evanescent wave, with a phase factor $\exp[j\omega(t - n\alpha x)]$, has a phase velocity $|\,1/n\alpha\,| < 1$ (as is appropriate for a wave carrying tachyon photons). With a velocity $\beta_E \simeq \beta_F \simeq 1/n\alpha$, our electrons feel a (nearly) standing photon wave, and exchange with it momentum but (practically) no energy quanta. With a velocity $\beta > 1/n\alpha$, they would feel a phase traveling backwards.

Fortunately, there exists a practical possibility for testing the properties of our tachyon photons: hyperfine transitions in an atomic beam traveling parallel to the x axis. In this case, the frequancy ω will be in the Hertzian

[5] It is possible to approximate a plane tachyon wave even much better by using two parallel reflecting surfaces rather than one; then, the real exponential is replaced by cosh or sinh functions assuming, respectively, the values 1 and 0 in the middle of the space between the surfaces.[22]

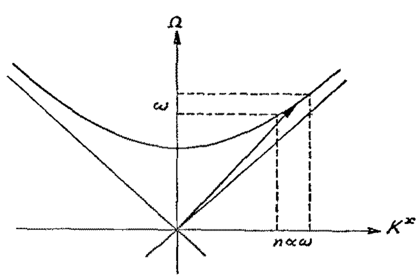

Fig. 7. First-order photoelectric effect on free electrons.

range, and the penetration depth of the wave will be such as to allow experiments.[24]

Let (Fig. 8) ω_F and ω_E denote the rest masses of atoms on the two levels we are considering ($\epsilon \equiv \omega_E - \omega_F > 0$), and ($\Omega_F$, $K_F \equiv \beta_F \Omega_F$) and ($\Omega_E$, $K_E \equiv \beta_E \Omega_E$) two points on the corresponding energy-momentum hyperbolas (F) and (E)

$$\Omega_F^2 - K_F^2 = \omega_F^2, \qquad \Omega_E^2 - K_E^2 = \omega_E^2 \tag{22}$$

The transition will occur if

$$\Omega_E - \Omega_F = \omega, \qquad K_E - K_F = n\alpha\omega \tag{23}$$

Geometrically, we are finding the four intersection points of the hyperbolas, one of which is translated by the vector ($n\alpha\omega$, ω); two of these points are at infinity and need not concern us.

From the preceding relations, we easily deduce a second-order equation in β_F (or in β_E) which, as $\omega, \epsilon \ll \omega_F, \omega_E$, whence $\beta_F \simeq \beta_E$, assumes the approximate form[25]

$$(n^2\alpha^2\omega^2 + \epsilon^2)\,\beta^2 - 2n\alpha\omega^2\beta + (\omega + \epsilon)(\omega - \epsilon) \simeq 0 \tag{24}$$

We are interested in a small value for β, that is, in cases such that

$$|\,\omega - \epsilon\,| \ll \omega, \epsilon;$$

then, the small root is $\beta \simeq (\omega - \epsilon)/n\alpha\omega$, whence

$$(\omega - \epsilon)/\omega \simeq n\alpha\beta \tag{25}$$

If $n\alpha \leqslant 1$, that is, if ordinary or limiting refraction occurs, (25) is the formula of the ordinary Doppler frequency shift. In our case $n\alpha > 1$, so we have a generalized Doppler shift such that the x component of the photon's group velocity is larger than 1—again a tachyon property, and an experimentally testable one. Imbert is presently planning experiments along these lines.

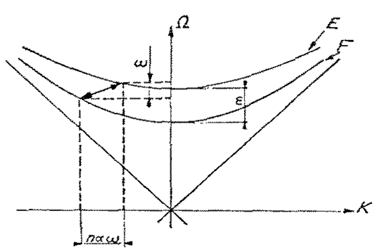

Fig. 8. Hyperfine transitions in an atomic beam.

In the Lorentzian frame of velocity $1/n\alpha$, the atoms exchange with a (standing) photon wave momentum quanta, but no energy quanta. In frames of velocity larger than $1/n\alpha$, the photon's phase travels backwards, and emission and absorption processes are exchanged; that is, $\Omega_F > \Omega_E$ although $\omega_F < \omega_E$.

4. THE QUESTION OF THE PHOTON'S ENERGY-MOMENTUM TENSOR

The classical energy-momentum tensor of the electromagnetic field is of course the Maxwell one, which is symmetric *in vacuo* with the well-known expression ($\alpha, \beta = 1, 2, 3$)

$$M^{ij} \equiv \begin{cases} M^{44} = w \equiv \frac{1}{2}(E^*E + H^*H) \\ M^{\alpha\beta} = \frac{1}{4}(E^{*\alpha}E^{\beta} + H^{*\alpha}H^{\beta} + \text{c.c.}) - w\delta^{\alpha\beta} \\ M^{\alpha 4} = M^{4\alpha} = -is^{\alpha} \equiv \frac{1}{4}(E^{*\gamma}H^{\beta} - E^{*\beta}H^{\gamma} + \text{c.c.}) \end{cases} \tag{26}$$

Inserting the expressions (10) and (11) with

$$\psi \equiv \psi_0 \exp[j\omega(t - n\alpha x) - (n^2\alpha^2 - 1)^{1/2}\,\omega y] \tag{27}$$

and with the definition

$$M \equiv E_0^{*z}E_0^{\,z} + H_0^{*z}H_0^{\,z}, \qquad N \equiv j(H_0^{*z}E_0^{\,z} - H_0^{\,z}E_0^{*z}) \tag{28}$$

we obtain[19]

$$M^{ij} \equiv \tfrac{1}{2}\exp[-2(n^2\alpha^2 - 1)^{1/2}\omega y]$$
$$\times \begin{pmatrix} -M & 0 & -(n^2\alpha^2 - 1)^{1/2}N & -in\alpha M \\ 0 & 0 & 0 & 0 \\ -(n^2\alpha^2 - 1)^{1/2}N & 0 & (1 - n^2\alpha^2)M & -in\alpha(n^2\alpha^2 - 1)^{1/2}N \\ -in\alpha M & 0 & -in\alpha(n^2\alpha^2 - 1)^{1/2}N & n^2\alpha^2 M \end{pmatrix} \tag{29}$$

The values $n\alpha M$ and $n\alpha(n^2\alpha^2 - 1)^{1/2} N$ [inside the last row of (29), *modulo* the factor $-i$] of the energy fluxes s^x and s^z have been experimentally tested in the sense that they are those which have yielded the correct explanations of, respectively, the Goos–Hänchen shift[19] by Kristoffel[17] and Renard[18] and of the new transverse Imbert shift[10] by Imbert[9]. However, when viewed as momentum densities, that is, in the fourth column rather than in the fourth row, these same expressions cannot be reconciled with what we have learned in Section 3 concerning the "tachyon photons." First, the Maxwell tensor yields a nonzero momentum density in the z direction (*with* *a nonzero integrated value*), while we know that the photon's momentum has no z component. Second, we see that ($-i$ times) the ratio of the momentum density in the x direction to the energy density is, according to (29), $1/n\alpha < 1$, while our previous study of the tachyon photons has yielded $k^x/\omega = n\alpha > 1$.

It is obvious that a symmetric energy-momentum tensor (i.e., essentially the Maxwell tensor in the vacuum) will never allow the momentum density M^{4z} to be zero if the energy flux M^{z4} is nonzero. Thus, we fall back on the conclusion that the noncollinear velocity and momentum of spinning particles require an asymmetric energy-momentum tensor *even in vacuum*.

It is also clear that if the Maxwell tensor (29) does not yield the right expressions for the momentum densities, it is because its relation to the momentum operator $j\partial$ (that is, also to the propagation vector $\mathbf{k}$ of plane or evanescent waves) is too indirect.[26] Thus, we are led to consider the asymmetric canonical energy-momentum tensor, that is, de Broglie's[2] tensor, for the photon field,

$$T^{ij} \equiv \tfrac{1}{8}A_l^*(\partial^i)\, B^{jl} + \text{c.c.} \tag{30}$$

It is the tensor canonically associated with the Lagrangian

$$L \equiv A_i^*[\partial_j]\, B^{ji} + \tfrac{1}{2}B_{ij}^*B^{ij} + \text{c.c.} \tag{31}$$

(whence the field equations $B^{ij} \equiv \partial^i A^j - \partial^j A^i$, $\partial_i B^{ij} = 0$, follow).

Using the transverse gauge in the rest frame of the refracting medium,

$$V = 0, \qquad \mathbf{A} = j\omega^{-1}\mathbf{E} \tag{32}$$

one easily finds[27] that the expression of the asymmetric de Broglie tensor inside Fresnel's evanescent wave is

$$T^{ij} = \tfrac{1}{2}\exp[-2(n^2\alpha^2 - 1)^{1/2}\omega y] \times \begin{pmatrix} -n^2\alpha^2 M & 0 & -n^2\alpha^2(n^2\alpha^2 - 1)^{1/2}N & -in\alpha M' \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -in\alpha M & 0 & -in\alpha(n^2\alpha^2 - 1)^{1/2}N & M' \end{pmatrix} \tag{33}$$

where, by definition,

$$M' \equiv E_0^{*z}E_0^{\ z} + (2n^2\alpha^2 - 1)\, H_0^{*z}H_0^{\ z} \tag{34}$$

Comparing (33) with (29), we see first that the energy fluxes are exactly the same as before:

$$T^{\alpha 4} = M^{\alpha 4}, \qquad \alpha = 1, 2, 3 \tag{35}$$

that is, that the de Broglie tensor also gives the experimental energy fluxes.

Second, inspecting the fourth column of (33), we find a zero momentum density in the z direction, in full accord with the fact that the photon's z-momentum component is zero: $k^z = 0$. Also, ($-i$ times) the ratio of the x component of the momentum density to the energy density is now found as $n\alpha > 1$, in full accord with the tachyon photon property that $k^x/\omega = n\alpha > 1$. Thus, everything is now all right with the momentum densities.

Finally, the intrinsic asymmetry of the de Broglie canonical energy-momentum tensor and its quite direct relation to the energy-momentum operator $j\partial^\lambda$ have settled all matters. Thus, in full conformity with the considerations in Section 1, asymmetry is a fundamental property of the energy-momentum tensor of spinning particles.

5. CONCLUSION

We do not pretend that the answers to all significant questions raised by the preceding considerations have been given. Among other things, a theoretical analysis of the linear and angular recoils of the reflecting surface is an interesting task.

What we do say is (1) the existence of an energy flux *through* the incidence planes has been experimentally demonstrated, in full accord with its theoretical prediction; and (2) there is no reasonable doubt that the very peculiar properties of "tachyon energy-momentum quanta" lying *inside* the incidence planes should come out experimentally as they are predicted.

Both statements taken together vindicate the claim of some theoretical physicists that under appropriate circumstances the velocity and momentum of spinning particles are noncollinear.

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