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Bohr's Discussion of the Fourth Uncertainty Relation Revisited

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Bohr's 1930 derivation of the uncertainty relation $c^2\,\delta m\,\delta t \geqslant \hbar$ bears a close relationship to Einstein's 1913 derivation of the "gravitational redshift" via the "equivalence principle." A rewording of Bohr's argument is presented here, not taking the last step of acceleration as "equivalent" to a uniform gravity field, thus yielding a derivation of the formula $c^2\,\delta m\,\delta t \geqslant \hbar$, avoiding Treder's 1971 objection.

In 1971 Treder[(1)] objected to Bohr's[(2)] 1930 derivation of the "fourth uncertainty relation" in the form $c^2\,\delta m\,\delta t \geqslant \hbar$ that the pan of his ideal balance need not be moved by the gravitational field, in which case the gravity dependence of the rate of clocks does not show up, which undermines the reasoning. However, argues Treder, any balance needs damping, entailing thermal degradation of an (unknown) energy δE related to the relaxation time δt by the formula $\delta E\,\delta t \geqslant \hbar$—a very direct argument overlooked by both Einstein and Bohr in their 1930 discussion!

This is a perfectly valid argument, but one displacing the burden of the proof from a fundamental to a side effect. Here we will vindicate Bohr's 1930 fundamentalistic argument by rewording it in terms of an acceleration (not a gravitational) field $\mathbf{g}$, thus conferring to it the same "universality" as Treder's argument (that is, independence with respect to the weighing mechanism). However, our idealization will be exactly opposite to Treder's: The restoring and the damping forces applied to the pan will be assumed vanishingly small, so that the experimental error δm on the measured mass m results in a constant unknown acceleration g conferred on the pan.

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After a time t the pan thus takes a velocity $v = gt$, resulting, according to the Lorentz formula $t' = (1-\beta^2)^{-1/2}\,(t - c^{-2}vx)$, at first order in $\beta = v/c$, in a variation in the rate of a clock $\delta t = c^{-2}vz = c^{-2}gtz$, whence

$$t^{-1}\,\delta t = c^{-2}g\,\delta z \tag{1}$$

Now, after time t, the acceleration g has conferred to the pan an unknown momentum $p_z = tg\,\delta m$ which, according to Heisenberg's "third uncertainty relation," is related to the error δz in the reading of the balance by the formula

$$tg\,\delta m\,\delta z \geqslant \hbar \tag{2}$$

Assuming that the error δz is the one exactly hiding all manifestations of the acceleration, δz must be the same in formulas (1) and (2), so that, eliminating it (that is, eliminating $tg\,\delta z$), we get

$$c^2\,\delta m\,\delta t \geqslant \hbar \tag{3}$$

All reference to the gravity field has been avoided, so that the reasoning holds whatever sort of mechanism is used in the balance.

The two assumptions that the restoring and damping forces are negligibly small ideally characterize a high-precision balance. Therefore this (modified) Bohr argument has the merit of connecting the "fourth uncertainly relation" to *essentials*: mass, time, and space measurements—and the role of *acceleration.*

As in Bohr's[2] initial presentation, there appears a marvelous "preestablished harmony" between the three formulas $E = h\nu$ or $p = h\kappa$ (quantal), $t^{-1}\,\delta t = c^{-2}g\,\delta z$ (pre-general relativity), and $E = c^2 m$ (special relativity). That the two former do yield an original derivation of the latter may look somewhat miraculous!

This expresses a tight binding between the fundamentals of the special relativity, the quantal, and the "pre-gravitation" Einstein 1913 heuristic theory[3] which may help us better understand the relation between the quantal and the gravitational theories.

In this respect, the interpretation of δt in the preceding reasoning may be of significance: It is an error in the proper time assignment of an event occurring on a accelerated body, the exact acceleration of which is unknown, as evaluated in an inertial frame.

Now, when taking the final step which has not been taken here, that is, thinking of a freely falling pan, the priority, so to speak, is reversed. A deep understanding of what is implied here may be a good physical hint in the direction of a truly satisfying "quantal theory of gravitation."

Incidentally, no reference whatever has been made to the opening and closing of a shutter, neither in our rewording of Bohr's argument, nor in our abridged presentation of Treder's.

REFERENCES

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