Foundations of Physics, Vol. 34, No. 11, November 2004 (© 2004)
DOI: 10.1007/s10701-004-1310-5

To Believe Or Not Believe In The A Potential, That's a Question. Flux Quantization in Autistic Magnets. Prediction of a New Effect

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Received August 25, 2004

Electromagnetic gauge as an integration condition *was my wording in previous publications. I argue here, on the examples of the Möllenstaedt-Bayh and Tonomura tests of the Ahraronov–Bohm (AB) effect, that not only the trapped flux* Φ *but also,* under the integration condition $\mathbf{A} \equiv \mathbf{0}$ *if* $\Phi = 0$, *the* local value *of the vector potential is measured.*

KEY WORDS: Aharonov–Bohm effect; autistic magnets; superconductivity; Ampère tension.

1. FOREWORD

This essay, together with a closely associated one, is dedicated to my good friend Franco Selleri in remembrance of our exchanges concerning fundamental physics. He is now busy scrutinizing the relativity theory, upholding unconventional testable ideas. Following his example, I allow myself to question here prevailing views pertianing to the interpretation of the electromagnetic potentials.

An analogy exists between Selleri's thoughts concerning the Sagnac optical effect and mine concerning the Aharonov–Bohm (A.B.) and Meissner electronic effects: both are expressed as a flux through a closed contour: Sagnac's via flux of the rotational velocity ω, A.B.'s and Meissner's via flux of the magnetic field $\mathbf{B}$; Larmor of course has likened in such a context ω and $\mathbf{B}$.

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2. INTRODUCTION

De Broglie,[1] in his history making work proposing the matter wave concept, expresses an electron wave's 4-frequency ($i, j, k, l = 1, 2, 3, 4$; $x^4 =$ ict; $U_i U^i = -c^2$) as the sum of a *mechanical* plus a *potential* contribution

$$k^i = \hbar^{-1}(m_\circ U^i - eA^i). \tag{1}$$

The A.B.[2] effect is thus postulated in both its electric and magnetic forms; so one wonders why 25 years elapsed between the two events!

The magnetic A.B. effect displays a physical influence of a curl less vector potential. But, as the closed linear integral $\Phi = \int \mathbf{A}\, dI$ expressing the trapped flux equals the surface integral $\Phi = \iint \mathbf{B}\, ds$, the opinion prevails that "it is the magnetic field **B** which, acting at a distance, causes the phase shift"—a *very metaphysical* statement! But if one thinks realistically of **A** as acting along the space–time trajectory one is lead, as Henneberger[3] among others has argued, to view **A** as a physical magnitude, its gauge being selected *as an integration condition*. This I have argued in previous works,[4–6] as the *fields* ($\mathbf{E}, \mathbf{B}$) are related to the *potentials* ($V, \mathbf{A}$) as *forces* are related to *energies or moments*. For example, an atomic mass defect is expressible in terms of the Coulomb potential. Similarly, momentum balance selects the magnetic gauge, as Konopinski[7] and others have argued.

So let us ponder the matter by revisiting the A.B. effect.

3. MÖLLENSTAEDT-BAYH'S TEST OF THE A.B. EFFECT

In Möllenstaedt's-Bayh's[8] device a solenoid of controllable flux Φ is inserted between beams issuing from an electronic biprism. I argue that, as the fringes pattern can be translated at will by varying Φ, something significant has been overlooked.

The A.B. effect consists in that, at a given point on the photograph, the phase shift $\Delta\varphi$ is proportional to the trapped flux Φ according to the gauge invariant formula

$$\Delta\varphi = -\hbar^{-1} e\mathbf{\Phi}; \tag{2}$$

All right. But *given* $\mathbf{\Phi}$, *according to de Broglie's formula* (1)*, under the integration condition*

$$\mathbf{A} \equiv \mathbf{0} \quad \text{iff} \quad \mathbf{\Phi} = 0 \tag{3}$$

the differential of the phase φ as a function of $\mathbf{r}$ along the fringes pattern displays (*beware ! one factor* $-e$ *per beam!*) *the semi quantum flux* $h/2e$ via

$$\hbar d\varphi = (m\Delta\mathbf{v} - 2e\mathbf{A})\cdot dI, \tag{4}$$

the collinear vectors $m\mathbf{\Delta v}$ and $-2e\mathbf{A}$ being contained in the picture's plane; *the vector potential* $\mathbf{A}$ *is thus measured, being expressed in the source adhering gauge.*

It then follows that $\mathbf{A}$ *exerts at any point in space a local effect, upstream no less than downstream.* This Tonomura[(9)] emphasizes p. 100 of his interesting book: see Fig. 80 and its legend.

So, *if the long solenoid were placed inside the interference pattern of a biprism, with the recording film just ahead and in direct contact with it, the pattern would be modified according to formula* (4)*;* this is a feasible test.

But a proof of this sort can already be found in Tonomura's[(9–11)] *experiments.*

4. TONOMURA'S HOLOGRAPHIC TESTS USING A TOROIDAL MAGNET

Tonomura superposes two beams issuing from a biprism, one shot through a small toroidal magnet, one bypassing it at a distance.

This generates a hologram, *the picture looking exactly as if the toroid were immersed in the fringes.* The "pictured ring" separates *external* and *internal straight fringes* of interfringe $m\mathbf{\Delta v}$ mutually shifted by $-2e\mathbf{A}$, *connected by wavy fringes of* Eq. (4) *with* $m\mathbf{\Delta v}$ *constant and* $-2e\mathbf{A}$ *radial*; these are ellipses with a focus at the ring's center.

So *the hologram looks exactly like a photograph printed on a film placed in contact with the magnet's front face, displaying the vector potential* $\mathbf{A}$ *tangent to the magnet's front face.*

Varying the flux Φ shifts the internal fringes, not the external ones which are the same as in absence of the magnet, because "how could a small toroidal magnet exert an influence at infinity?" The displayed ratio $-2e\mathbf{A}/m\Delta v$ yields a measurement of the vector potential $\mathbf{A}$ as a function of Φ, independent of Δv.

This remains true in the limit $\mathbf{\Delta}v = 0$. A Tonomura photograph obtained via electron interference microscopy shows circular fringes, the phase of wich displays the *potential space frequency* via the equation $\hbar\varphi = -e\int \mathbf{A}\,dr$ yielding a measurement of $\mathbf{A}$ as tangent to the magnet's front face. This *open* line integral yields half the A.B. effect; but adding a similar one in the plane touching the magnet's rear face restores the full

A.B. effect. *The factor 2 thus stepping in produces the semi flux quantum* $h/2e$.

That the vector potential does exert *a local physical action* is evidenced in cases where the number of trapped semi-fluxons is odd, and the magnet's inner radius is smaller than the interfringe: *the "ring's inside being then black" all electrons are forced to bypass the magnet* by virtue of the Born–Jordan probability rules. *This is a* c^{-2} *group velocity effect distinct from the phase velocity A.B. effect* which is discussed in the following paper.

In this series of Tonomura tests the flux Φ being not strictly trapped was not quantized.

5. FLUX QUANTIZATION IN AUTISTIC MAGNETS

Let be termed(12) *autistic* a magnet *completely* trapping its flux, as does an infinitely long straight magnet or a toroidal one. I argued, via resonance of the evanescent electronic wave surrounding the magnet, that its trapped flux Φ is quantized in h/e units, "this rendering the A.B. effect unobservable". But later, in association with Vigoureux,(13) we found that in the case of a cylindrical magnet the integer and half integer Bessel functions entail existence of *two intercalated flux ladders, whence flux quantization in h/2e units.*

To get a truly autistic magnet Tonomura(10,11) resorted to superconduction, this bringing in Cooper pairs and the Meissner effect; his toroidal magnet was coated with a superconducting layer. Then, Φ being *truly* quantized in $h/2e$ units, the A.B. effect is evidenced by contrasting the cases of even or odd numbers of trapped semi-fluxons $h/2e$: the fringes inside the ring are either the same or opposite to the outside ones, according as the number of trapped semi-fluxons is even or odd.

As each $h/2e$ quantum added or substracted in Φ translates the inside fringes by half an interfringe, one sees by likening the A.B. closed contour to two open ones, one before one after the magnet, "the semi-fluxon working again its miracle".

In this Tonomura series of tests no wavy fringes appear on the ring's surface. This may be due to the fact that the magnet's coated front face was not a smooth plane.

6. FLUX QUANTIZATION VIA GEOMETRY

Idealizing the "autistic magnet" as a closed (not necessarily circular) filament carrying per line element $dI \equiv d_1 r$ a magnetic moment $\mathbf{\Phi}\, dI$, we express the source adhering vector potential created at $x_2 \equiv x_1 + \mathbf{r}$ as

$$\mathbf{A}(\mathbf{r}) = \Phi \int r^{-3}\mathbf{r}_{-} dI. \tag{5}$$

Then, Ω denoting the solid angle through which the magnet is seen from x_2, the *magnetic potential action* of a flying electron comes out as

$$-e \int \mathbf{A}\, d_2\mathbf{r} = -e\Phi \int \int r^{-3}\mathbf{r}\,[d_1\mathbf{r}_\times d_2\mathbf{r}] = -e\Phi\Delta\Omega. \tag{6}$$

If the observation point x_2 is displaced along a closed circuit embracing the magnet the solid angle Ω varies by multiples of 4π and the action by multiples of h. *Again, the magnetic flux quantum comes out as $h/2e$.*

If the observation point is displaced from minus to plus infinity along a straight line through the magnet Ω varies by plus or minus 4π; but it does not change any more if the contour is closed outside the magnet. This confirms that in Tonomura's pictures the external fringes are the same as in absence of the magnet.

So, the *longitudinal phase shift* $\Delta\varphi$ at the magnet's center is caused by the *magnetic action integral* $-e\int \mathbf{A}\, dI$ along the central ray. The *transverse interfringe shift* is due to the *transverse mechanical action integral* $m \int \boldsymbol{\Delta} v \cdot dI$.

Finally, *the sacrosanct Φ dependence is recovered at the picture's center, thanks again to the factor 2 in $-2e\mathbf{A}$. Each* of the radial contour integrals $\int (m\boldsymbol{\Delta}\mathbf{v} - 2e\mathbf{A}) \cdot \boldsymbol{dI}$ "across the ring" yields the canonical A.B. closed contour shift.

7. MEISSNER EFFECT AND AMPERE'S STRESS TENSION

Consider[4] two interlaced filaments, the one a superconducting wire of line element $\boldsymbol{dI}_\mathbf{c}$ carrying Cooper pairs with the intensity $I_c \equiv -2\nu e$, the other an autistic magnet of line element $\boldsymbol{dI_m}$ trapping a flux $\Phi_m = -n_m h/2e$. Denoting by n the *algebraic* number of twists, we express *the system's quantized mutual energy* as

$$W = I_c\Phi_m \int \int r^{-3}\mathbf{r}[\boldsymbol{dI}_{\mathbf{c}_\times} \boldsymbol{dI_m}] = nn_m h\nu. \tag{7}$$

The Meissner effect consists in that the total flux embraced by the current is zero : $\Phi_m + \Phi_c = 0$: *the sum of the mutual energy W and the current's self energy is zero.* As Tonomura[9] puts it p. 133 *gauge symmetry* *is broken in superconductivity.* Both vectors **A** and **v** are curlless, and the supercurrent flows at the wire's surface with a velocity $\mathbf{v} = -e/m\mathbf{A}$; so **A** *comes out as expressed in the source adhering gauge.*

It is noteworthy(14) that *the current's self energy equals the flowing electrons' kinetic energy,* as is seen via $I \equiv -2\nu e$ and

$$\nu \; m\mathbf{v} \cdot \boldsymbol{dv} = -e\mathbf{A} \cdot \boldsymbol{dl}. \tag{8}$$

So the running electrons' centrifugal force causes along the wire a stress tension

$$\mathbf{T} \equiv I\mathbf{A}. \tag{9}$$

8. MACROSCOPIC AMPERE TENSION AND VASCHY'S PARADOX

The *"Ampère tension"* of expression (9) is easily shown(15) to be integrally equivalent to the *Weber action–reaction force* between two current elements

$$d^2\mathbf{F} = I_1 I_2 r^{-2}\mathbf{r}(\boldsymbol{dI}_1 \cdot \boldsymbol{dI}_2). \tag{10}$$

This, contrary to a prevailing(16) *feeling, contradicts not the Laplace or Grassmann transverse force concept.* Consider(4) the *small* area inside the contour generated by a *local* deformation of the circuit between two nearby points P and Q. The *virtual work of the Grassmann force* is $dW = I \int \boldsymbol{A} \cdot \boldsymbol{dI} = I \int\int \boldsymbol{B} \cdot \boldsymbol{ds}$, Q.E.D.

Ever since the historical Ampère-La Rive experiment quite a few other ones have aimed at testing a *local* repulsive stress tension appearing if a circuit is cut. Among these Saumont's(17) is especially clear and persuasive: L denoting the dimensionless self-induction factor, T's expression comes out as $T = LI^2$ emu.

It so happens that a 1890 *Treatise* by Vaschy(18) contains the surprising statement that "no mutual energy exists between a circuit and a permanent magnet"; *do not a circuit and a magnet interact?* Vaschy argued from *the fact* that *no e.m.f. appears when a circuit's intensity is varied in presence of a permanent magnet.* But what this proves(14) is that *the sum of the electromagnetic mutual energy plus a mechanical constraining energy is zero. What is this energy, "that's the question".*

Read on the formula the answer is: *the Ampère–tension potential energy* $I\mathbf{A} \cdot \boldsymbol{dl}$ *per line element*—this confirming the previous interpretation of the Meissner effect.

In 1914 Blondel[19], using an *ad hoc* device, claimed he had vindicated Vaschy's statement; in his setup a pulley rolled in or out a current carrying wire in presence of a permanent magnet; no positive or negative e.m.f. showed up. *But this device is a "generator-or-motor"* of the Barlow, Gramme, unipolar rotor family. *Had Blondel measured the torque on his rotating pulley he would have retrieved the missing energy—and evidenced the Ampère stress tension* $\mathbf{T} = I\mathbf{A}$.

9. WEBER'S ACTION AT A DISTANCE BETWEEN CURRENT ELEMENTS AND RELATIVISTIC COVARIANCE

Please think of this. Two conduction electrons distant by r in a straight wire feel the repulsive Coulomb force expressed in their rest frame as $r_{\circ}^{-2}e^2$, *plus a velocity dependent repulsive force due to the Lorentz transform to the conductor's frame.* As the mutual force between the two electrons thus is $r^{-2}e^2(1+\beta^2)$ the expression (10) of the repulsive Weber force along a straight conductor can be derived.

But what in the general case? An isomorphism exists[20] between the three-dimensional magnetostatics of conducting filaments and the four-dimensional Wheeler–Feynman[21] electrodynamics of point charges; it stems from the correspondence recipe between *Weber's mutual energy* and *Wheeler–Feynman's mutual action*

$$r^{-1} I I' \boldsymbol{dI} \cdot \boldsymbol{dI}' \_ \delta(r^2) e e' dx^i \, dx'_i; \tag{11}$$

Wheeler–Feynman's far action electrodynamics *transposes* Weber's magnetostatics of currents. Both are integrally equivalent to the conventional ones using the transverse Grassmann or Lorentz force.

10. CONCLUDING REMARKS

The ideas upheld here conform to those previously worded[4–6] *"electromagnetic gauge as an integration condition"*. The argument was that the relation between *fields* and *potentials* parallels that between *forces* and *moments or energies,* which relation is *an integration.* Louis de Broglie[22] very often argued along such lines.

As my good friend Franco Selleri has recently played in physics the time honoured game of carefully wording bold testable hypotheses, all right, I follow his example.

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