*Foundations of Physics, Vol. 34, No. 12, December 2004 (© 2004)*
DOI: 10.1007/s10701-004-1627-0

# To Believe or Not Believe in the 4-Potential, That's a Question. The Electric Helmholtz–Mikhailov Effect and its Magnetic Analog

**O. Costa De Beauregard**[1]

*Received August 25, 2004*

*Helmholtz' electrically induced extra mass inside a charged hollow sphere, recently evidenced by Mikhailov, is analogous to Mach's inertial mass. Existence of a corresponding magnetically induced extra mass in an electron flying around an "autistic magnet" is derived. The overall electro-magnetic effect can be covariantly expressed.*

**KEY WORDS:** energy-mass and inertia gravity equivalences; electromagnetically induced extra inertia by a fieldless 4-potential.

## 1. FOREWORD

This essay, together with the previous closely associated one, is dedicated to my good friend in remembrance of our many exchanges concerning the fundamentals of physics.

## 2. MIKHAILOV'S MEASUREMENTS OF AN ELECTRON'S INDUCED EXTRA MASS INSIDE A HOLLOW CHARGED SPHERE

In 1872 Helmholtz(1) deduced from Weber's(2) electrodynamics, in which $c$ is defined as the ratio of the emu and esu units systems, that a "potential extra mass" $-c^{-2}eV$ is induced in (let us say) an electron moving inside a charged hollow sphere enclosing a fieldless potential $V = Q/R$. Disbelieving this result he used it as an argument for rejecting

[1]Fondation Louis de Broglie, 23 rue Marsoulan, 75012 Paris; e-mail: fargue@de.ensmp.fr

Weber's theory. But two recent measurements by Michailov[3] display the phenomenon, which is derivable either via Weber's[4] or relativistic[5] electrodynamics. Anyhow there is a consensus concerning the analogous effect in the field of gravity, namely "Mach's conjecture".

## 3. GRAVITATIONAL DIGRESSION: MACH'S CONJECTURE

Mach[6] argued that a mass point's inertial mass $m_{\mathrm{i}}$ is *induced* in it by the cosmologic potential $U_0$, namely $U_0 = GM/R$ in the "sphere of fixed stars" model; Sciama[7] turned this conjecture into an algebraic formula. Following recent papers by Woodward–Mahood[8] and myself,[9] let Mach's conjecture be recalled in terms of the two premises of the Galileo–Newton equivalence $m_{\mathrm{i}} = m_g$ of inertial and gravitational masses, and of the Weber–Einstein equivalence $W = c^2 m$ of energy and mass.

Inside the *uniform cosmological potential* $U_0$ a mass point of *gravitational mass* $m_g$ has *a potential energy* $U_0 m_g$ whence, via *mass-energy equivalence,* an *induced mass* $c^{-2}U_0 m_g$. This, according to Mach, is its inertial mass $m_{\mathrm{i}}$. Then, via the inertia-gravity equivalence, one gets[8,9] $U_0 = c^2$; and of course $GM/c^2R \approx 1$ is a formula valid in most cosmological models.

*But this ends not the story.* If a gravitational potential confers an inertial mass to a mass point, then a test particle (say the planet Mercury) orbiting a strong gravity source (say the Sun) of potential $U_S$ must, like Sommerfeld's electron orbiting the proton, pick up an extra mass $c^{-2}U_S m_g$; Tisserand,[10] using a transposition of Weber's electrodynamics, proposed this in 1872. So, on the whole, Mercury's *effective inertial mass* must read

$$m_{\mathrm{i}} = \left(1 + U_{\mathrm{S}}/c^2\right) m_g. \tag{1}$$

For some reason this formula yields[4] $1/3$ of the observed perihelion advance.

*What* if the test particle's mass is not negligibly small, so that the source recoils? De Broglie[11] and Lucas[10] show, via barycenter conservation, that *the overall mutual potential mass is distributed inversely to the bare masses.* Essentially, their argument consists of this. The classical formulas $\sum m\mathbf{r} = \mathbf{0}$ and $\sum m\mathbf{v} = \mathbf{0}$, valid in the barycenters rest frame, say that the particles, displacements and velocities are inversely proportional to their masses. As the kinetic energies—$\frac{1}{z}m\mathbf{v} \cdot \mathbf{v}$ are naturally thought of as localized in the particles so are also, according to mass-energy equivalence, the contributions of the potential energy.

In the case of just two interacting particles, one very heavy, one very light, the light particle picks up almost all the potential mass. This is Mach's statement, yielding a derivation of mass-energy equivalence from purely mechanical arguments. It likens the resistance to accelerating a body to "action-reaction with the distant stars"—*a momentous conclusion* indeed!

## 4. RETURNING TO ELECTRODYNAMICS

In Sommerfeld's hydrogen atom model the electron's kinetic energy equals minus its potential energy expressed in the Coulomb gauge. So, via Einstein's mass-energy equivalence, *the system's potential mass resides entirely in the orbiting electron,* in accord with de Broglie's and Lucas' statement. Similar to the perihelion advance, the periproton advance thus evidences the *electrostatically induced extra mass.* It is then likely that, similar to Mach's induced inertia, the Helmholtz one exists for an electron accelerated inside a hollow charged sphere enclosing a Coulomb potential $V = Q/R$.

This can be confirmed via action–reaction. An electron, accelerated by any means (say, the force of gravity) inside the sphere from zero to a small velocity $\mathbf{v}$ confers to the sphere a momentum $-\mathrm{e}\mathrm{R}^{-1}\mathrm{Q}\mathbf{v}$; then *the induced extra mass* $-c^{-2}eV$ *expresses the reaction it feels.*

As this has to do with barycenter conservation let the matter be looked at from the other side. Inside a charged hollow sphere accelerated at $\mathbf{g}$ the 4-potential "rotates hyperbolically à la Minkowski"; so, according to the formula $\mathbf{E} = \mathrm{c}^{-1}\partial_t\mathbf{A}$, *the sphere contains a uniform electric field* $\mathbf{E} = \mathrm{c}^{-2}\mathrm{V}\mathbf{g}$. Via inertia-gravity equivalence the same holds if the sphere is fixed in a gravity field $\mathbf{g}$ -say "if it rests on the laboratory floor". Inside such a sphere a test electron feels the force $-\mathrm{e}\mathbf{E} = -\mathrm{c}^{-2}\mathrm{eV}\mathbf{g}$; *its induced extra mass is thus "weighed" as equal to the overall potential mass,* Q.E.D.

But, as such electric style thinking is unfitting in magnetic cases, let us tackle a magnetic example.

## 5. EXTENDING THE INVESTIGATION TO MAGNETODYNAMICS

A test electron flying at velocity $\mathbf{v}$ in presence of a heavy toroidal magnet moves inside a curlless vector potential $\mathbf{A}$, the gradient of a multivalued function. Denoting by $\mathbf{dM} = \Phi\mathbf{dl}$ a (not necessarily circular) *curvilinear toroidal magnets* elementary moment, we express the systems mutual energy as

$$W = -e\Phi \int r^{-3}[\mathbf{v} \times \mathbf{r}] \cdot \mathbf{dl}$$
$$= -e\Phi \int r^{-3}[\mathbf{r} \times \mathbf{dl}] \cdot \mathbf{v} \tag{2}$$

evidencing the electrons magnetic field **H** and the magnets vector potential **A** via

$$W = \Phi \int \mathbf{H} \cdot \mathbf{dl} = -e\mathbf{A} \cdot \mathbf{v}, \tag{3}$$

$W$ is thus expressed as residing either in the magnet or in the electron. In the latter case, the one physically making sense, $-c^{-2}e\mathbf{A} \cdot \mathbf{v}$ is the magnetic analog of the Helmholtz extra-mass. It also is the one displayed in the Meissner effect as discussed in our previous paper.

As for the induced moments, we consider the restricted case where the magnet is circular and the electron flies along its axis; then $\mathbf{v} \cdot \mathbf{dl_m} = 0$. $\mathbf{E} = \mathbf{v}\_\mathbf{H}$ denoting the flying electrons, electric field we can write momentum balance in the form

$$\mathbf{P} = c^{-2}e\Phi \int \mathbf{E} \times \mathbf{dl} = -c^{-2}e(\mathbf{A} \cdot \mathbf{v})\mathbf{v} \tag{4}$$

the magnet's moment being Poyinting style, and the electron's one transposing magnetically the Helmholtz electric one.

As **A** and **v** are then collinear $\mathbf{P} = -\beta^2 e\mathbf{A}$. *This expresses a group velocity effect distinct from the phase velocity A.B. effect.*

## 6. COVARIANT EXPRESSION OF THE ELECTROMAGNETICALLY INDUCED MOMENTUM-ENERGY

For issuing a *covariant electromagnetic* statement two remarks are in order: *First, Action–reaction is a key ingredient;* Second, *A curlless 4-potential induces an extra 4-momentum.* The guess then is that *the effective rest mass of a test electron including an induced contribution* should be expressed as

$$m = \mu - c^{-2}eA_k U^k \quad \text{with} \quad U_k U^k = -c^2. \tag{5}$$

*Is this compatible with acceleration of the electron by an electromagnetic field* $H^{ij}$*?*

*It is.* The Lorentz equation of motion of an electron, or equivalently *its spacetime trajectory,* can be derived from the extremed action recipe

$$0 = \delta \int (\mu U_{\mathrm{i}} - \mathrm{e}A_{\mathrm{i}})\mathrm{dx}^{\mathrm{i}} = \delta \int mU_{\mathrm{i}}\mathrm{d}x^{\mathrm{i}}, \tag{6}$$

where the second expression is obtained by multiplying $-\mathrm{e}A_{\mathrm{i}}$ by $U_{\mathrm{k}}U^{\mathrm{k}} = -c^2$ and exchanging indexes between the 4-vectors $U^{\mathrm{i}}$ and $dx^{\mathrm{i}}$ which are collinear if the path integral is the mechanical trajectory.

So, *whenever the 4-potential contains a curlless contribution, this acts as a sui generis 4-force modifying the rest mass by inducing a contribution* $-\mathrm{c}^{-2}\mathrm{e}\ A_{\mathrm{k}}U^{\mathrm{k}}$. This formalizes an action–reaction effect between a test electron and the sources of a curl-less 4-potential.

More generally, the force accelerating an electron can be severed in a conventional Lorentz ponderomotive force plus an extra inertial force of the kind just discussed.

This means that, *underlying the Aharonov-Bohm phase velocity effect, there exists a group velocity effect of order* $c^{-2}$. The extremed action recipe formalizes both, as integrated either along the $\mu U^{\mathrm{i}} - eA^{\mathrm{i}}$ lines or along the $\mu U^{\mathrm{i}}$ lines. As was argued in our previous paper, *this effect can be tested.*

*This group velocity effect shows up as the Meissner effect* in a superconducting wire. The Meissner effect selects the gauge via the formula $m\mathbf{v} = \mathrm{e}\mathbf{A}$: *the electrons total kinetic energy thus equals*[14] *minus the interaction energy.*

## 7. CONCLUDING REMARKS

Mikhailov's measurements, confirming a Helmholtz 1872 calculation and vindicating statements by Assis and myself, are of significance. An analogous magnetic effect, concerning an electron accelerated in presence of an "autistic magnet" exists also. Covariantly expressed, *the overall electromagnetic phenomenon* consists of an extra rest mass induced in an electron by action–reaction with the sources of a fieldless 4-potential. *The 4-potential then shows up as a physical magnitude, its gauge being the source adhering one.*

## REFERENCES

1. V. F. Mikhailov, *Ann. Fond. L. de Broglie* **24,** 161 (1999) and **28**, 231 (2003).
2. H. von, Helmholtz, *Phil. Mag.* **44,** 530 (1872).
3. W. Weber, *Werke* (Springer, Berlin 1892–1894).

4. A. K. T. Assis, *Weber's Electrodynamics* (Kluwer, Dordrecht. See p. 193 and p. 203, 1994).
5. O. Costa de Beauregard, "Electromagnetic Gauge as Integration Condition", in *Advanced Electrodynamics,* T. W. Barrett and D. M. Grimes eds. (World Scientific, Singapore, 1995) pp. 77–104.
6. O. Costa de Beauregard, *Phys. Essays* **10,** 492, 646 (1997).
7. E. Mach, *The Science of Mechanics* (Open Court, La Salle, 1926).
8. D. W. Sciama, *Month. Not. Roy. Astron. Soc.* **113,** 34 (1953).
9. J. F. Woodward and T. Mahood, *Found. Phys.* **29,** 899 (1999).
10. M. F. Tisserand, *C. R. Ac. Sci.* **75,** 760 (1872).
11. O. Costa de Beauregard, *Found Phys. Lett.* **13,** 395 (2000).
12. L. de Broglie, *C. R. Ac. Sci.* **275B,** 899 (1972).
13. R. Lucas, *C. R. Ac. Sci.* **282B,** 43 (1976).
14. O. Costa de Beauregard, *Ann. Fond. L. de Broglie* **28,** 77 (2003).