Experimental Evidence of the Einstein Paradox.

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This paper emphasizes a very specific difference between the paleo-quantal and the neo-quantal predictions pertaining to the correlation of polarizations of the photon pairs issuing from a cascade transition, the recent experiments (1) being definitely in favour of the latter. In this light, three definite statements pertaining to the nature of the Einstein (2) paradox will be drawn.

The neo-quantal probabilities of the yes-yes and no-no, yes-no and no-yes answers to the question put to the photons (flying in opposite directions along an axis x) by a pair of linear analysers with relative angle α are

$$\langle 1, 1\rangle = \langle 0, 0\rangle = \tfrac{1}{2}\cos^2\alpha\,, \qquad \langle 1, 0\rangle = \langle 0, 1\rangle = \tfrac{1}{2}\sin^2\alpha\,, \tag{1}$$

with the 0-1-0 cascades, and

$$\langle 1, 1\rangle = \langle 0, 0\rangle = \tfrac{1}{2}\sin^2\alpha\,, \qquad \langle 1, 0\rangle = \langle 0, 1\rangle = \tfrac{1}{2}\cos^2\alpha\,, \tag{2}$$

with the 1-1-0 cascades.

First we derive simply these expressions from the neoquantal rule of squaring the absolute value of the sum of partial amplitudes, using in succession two orthogonal bases: the circular, and two perpendicular linear, polarizations.

From the two pure helicity states $L_a L_b$ and $R_a R_b$ of the photon pair we build the parity invariant orthogonal states

$$L_a L_b + R_a R_b = E^y_a E^y_b + E^z_a E^z_b\,, \tag{3}$$

$$L_a L_b - R_a R_b = i[E^y_a E^z_b - E^z_a E^y_b]\,, \tag{4}$$

(1) S. J. FREEDMAN and J. F. CLAUSER: *Phys. Rev. Lett.*, **28**, 938 (1972); J. F. CLAUSER: *Phys. Rev. Lett.*, **36**, 1223 (1976); E. FRY and R. C. THOMSON: *Phys. Rev. Lett.*, **37**, 465 (1976).

(2) A. EINSTEIN, in *Rapports et Discussions du V Conseil Solvay* (Paris, 1928), p. 253.

the former appearing in the 0-1-0 and the latter in the 1-1-0 cascades; the y and z axes, together with x, define a Cartesian tripod. We present the reasoning with formula (3), but it would be quite similar to formula (4).

First we use the basis made of circular polarizations. Rotating by ΔA the analyser on the a-beam changes the relative phase by (say) $+\Delta A$ for the L pair, and (then) $-\Delta A$ for the R pair. Similarly, rotating by ΔB the analyser on the b-beam changes the phase by $-\Delta B$ for the L pair and $+\Delta B$ for the R pair. Thus setting

$$\alpha \equiv B - A\,, \tag{5}$$

we obtain $\exp[i\alpha]$ and $\exp[-i\alpha]$ as the partial (orthogonal) amplitudes. The neoquantal rule then gives (1) in the form

$$\langle 1, 1\rangle = \langle 0, 0\rangle = \tfrac{1}{8}|\exp[i\alpha] + \exp[-i\alpha]|^2\,, \quad \langle 1, 0\rangle = \langle 0, 1\rangle = \tfrac{1}{8}|\exp[i\alpha] - \exp[-i\alpha]|^2\,. \tag{6}$$

Second we use the basis made of two perpendicular linear polarizations. The transition amplitudes (as deduced from the classical transition probabilities) are in this case $\cos A \cos B$ and $\sin A \sin B$ for the 0,0 and 1,1 answers, $\cos A \sin B$ and $\sin A \cos B$ for the 1,0 and 0,1 answers. Together with formula (5) the neoquantal rule now yields (1) in the form

$$\begin{cases} \langle 1, 1\rangle = \langle 0, 0\rangle = \tfrac{1}{2}(\cos A \cos B + \sin A \sin B)^2\,, \\ \langle 1, 0\rangle = \langle 0, 1\rangle = \tfrac{1}{2}(\cos A \sin B - \sin A \cos B)^2\,. \end{cases} \tag{7}$$

Expanding formulae (6) and (7) will yield diagonal and off-diagonal terms. The sum of the diagonal terms is the paleoquantal expression for the probabilities at stake, obeying the old law of addition of partial probabilities. The sum of the off-diagonal terms is the neoquantal phase-dependent correction to these. With the circular polarizations basis we thus obtain

$$\langle 1, 1\rangle = \langle 0, 0\rangle = \tfrac{1}{4}(1 + \cos 2\alpha), \qquad \langle 1, 0\rangle = \langle 0, 1\rangle = \tfrac{1}{4}(1 - \cos 2\alpha) \tag{8}$$

and, with the (perpendicular) linear-polarization basis,

$$\begin{cases} \langle 1, 1\rangle = \langle 0, 0\rangle = \tfrac{1}{2}(\cos^2 A \cos^2 B + \sin^2 A \sin^2 B) + \tfrac{1}{4}\sin 2A \sin 2B\,, \\ \langle 1, 0\rangle = \langle 0, 1\rangle = \tfrac{1}{2}(\cos^2 A \sin^2 B + \cos^2 A \sin^2 B) - \tfrac{1}{4}\sin 2A \sin 2B\,. \end{cases} \tag{9}$$

Contrary to (1), (6) or (7), neither the paleoquantal probabilities (first terms in eqs. (8) or (9)) nor the neoquantal corrections (second terms) are basis invariant.

They are even not rationally invariant around the x-axis in (9). Therefore the paleoquantal physicist would have taken in (9) the rotational mean value. This we do most easily by writing

$$\tfrac{1}{4}\sin 2A \sin 2B = \tfrac{1}{8}\{\cos 2\alpha - \cos 2(A + B)\} \simeq \tfrac{1}{8}\cos 2\alpha \tag{10}$$

($\cos 2(A + B)$ being 0 in the mean). Therefore the paleoquantal probabilities, when using the linear polarizations, were

$$\langle 1, 1\rangle_0 = \langle 0, 0\rangle_0 = \tfrac{1}{4} + \tfrac{1}{8}\cos 2\alpha\,, \qquad \langle 1, 0\rangle_0 = \langle 0, 1\rangle_0 = \tfrac{1}{4} - \tfrac{1}{8}\cos 2\alpha \tag{11}$$

and thus different from the neoquantal ones (8).

As the experiments [1] definitely favour the neoquantal probabilities (1) and completely rule out the paleoquantal ones, this directly entails the following three general and far-reaching statements:

I) The photons a and b do *not possess* a polarization of their own when leaving the source C, but *borrow* one *later*, through interaction with the analysers L and N.

To see this quite crudely consider the case $\alpha = \pi/2$, where the neoquantal probability $\langle 1, 1\rangle$ has the value 0. If the photons did possess circular polarizations (of same helicity) when leaving C the joint transition probability would be the paleoquantal one, $\frac{1}{4}$ in this case. If they did possess (parallel) linear polarizations with (necessarily) random orientations, the joint transition probability would be the paleoquantal one, $\frac{1}{8}$ in this case.

Anyhow, according to the (paleoquantal) law of addition of partial probabilities, the sub-ensemble of photon pairs with (parallel) linear polarizations parallel to either the y- or the z-axis would be of measure 0. Quite the contrary, the photon pairs with this property are in fact the *ensemble of all photon pairs* in this experimental situation!

II) As a corollary, in our gambling game, the dye is cast *not* at C but *later*, where and when a measurement is made, at L and/or N.

This is the Einstein [2] *paradox*, better known as the Einstein-Podolsky-Rosen [3] paradox, which EINSTEIN [4], SCHRÖDINGER [5], DE BROGLIE [6], and others [7] *a priori* refused, but which today experiments [1] do show is really [8] there.

III) The correlation *existing* between the measurements at L and N is *not* drawn in space-time along the spacelike vector LN (which is physically empty). It is drawn along the Feynman style zigzag made of the two timelike vectors LC and CN, (which are physically occupied), once towards the past, once towards the future, as shown quite clearly by a calculation [9].

This is the interpretation of the Einstein paradox I have proposed quite a few times since 1962 [10], and towards which other authors [11] are more or less coming.

IV) The preceding formulae are specifications of a general neoquantal scheme which, though implicit in most writings on the neoquantal correlation, has been made explicitly first by GARUCCIO and SELLERI [12] of whom I present here the idea in my own way.

According to the neoquantal law of addition of partial amplitudes, two correlated subsystems φ and ψ are described by a pure state Ψ expanded in the form

$$|\Psi\rangle = \sum c_i |\varphi_i\rangle |\psi_i\rangle \tag{12}$$

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(3) A. EINSTEIN, B. PODOLSKY and N. ROSEN: *Phys. Rev.*, **47**, 777 (1935).

(4) A. EINSTEIN: in *Einstein Philosopher Scientist*, edited by P. A. SCHILPP (Evanston, Ill., 1949), p. 85, 683.

(5) E. SCHRÖDINGER: *Naturwiss.*, **23**, 807, 823, 844 (1935). See, p. 845.

(6) L. DE BROGLIE: *Une tentative d'interprétation causale et non linéaire de la mécanique ondulatoire* (Paris, 1956), p. 73.

(7) M. RENNINGER: *Physik*, **158**, 417 (1970); *Phys. Zeits.*, **136**, 251 (1963).

(8) *Paradox*: a surprising but perhaps true statement (sense No. 1 in all dictionaries). Copernicus' heliocentrism has been a paradox.

(9) O. COSTA DE BEAUREGARD: *Phys. Lett.* 60A, 93 (1977) ~~(in press)~~. See also, *Compt. Rend.* 283A, 1003 (1976).

(10) O. COSTA DE BEAUREGARD: *Rev. Intern. Philos.*, **61-62**, 1 (1962); *Dialectica*, **19**, 1 (1962); **19**, 280 (1965); in *Proceedings of the International Conference on Thermodynamics*, edited by P. T. LANDSBERG (London, 1970), p. 540; *Found. Phys.*, **6**, 539 (1976).

(11) H. P. STAPP: *Nuovo Cimento*, **29** B, 270 (1975); J. S. BELL: *Epist. Lett.*, **9**, 11 (1976); W. C. DAVIDON: preprint.

(12) A. GARUCCIO and F. SELLERI: preprint.

with

$$w_j \equiv c_j^* c_j\,, \quad \sum w_j = 1\,. \tag{13}$$

The (basis invariant) correlated mean value of two physical magnitudes A and B respectively measurable on φ and ψ is

$$\langle A, B\rangle = \sum\sum c_i^* c_j \langle\varphi_i|A|\varphi_j\rangle\langle\psi_i|B|\psi_j\rangle\,. \tag{14}$$

Setting

$$\langle A_i\rangle \equiv \langle\varphi_i|A|\varphi_i\rangle\,, \quad \langle B_i\rangle \equiv \langle\psi_i|B|\psi_i\rangle \tag{15}$$

and separating the diagonal and off-diagonal terms, we obtain

$$\langle A, B\rangle = \langle A, B\rangle_0 + \Delta\langle A, B\rangle\,, \tag{16}$$

where $\langle A, B\rangle_0$ is the (noninvariant) paleoquantal correlated mean value

$$\langle A, B\rangle_0 = \sum w_i \langle A_i\rangle\langle B_i\rangle \tag{17}$$

obeying the law of addition of partial probabilities, and implying separate statistics on φ and ψ (possibly « local hidden variables »), while

$$\Delta\langle A, B\rangle = \tfrac{1}{2}\sum_{i\neq j} c_i^* c_j \langle\varphi_i|A|\varphi_j\rangle\langle\psi_i|B|\psi_j\rangle + \text{c.c.} \tag{18}$$

is the (noninvariant) neoquantal correction, implying the phase differences.

A sufficient condition for making $\Delta\langle A, B\rangle$ zero is the diagonalization of at least one of the operators A or B. With such a particular basis the statistics will be that of a classical mixture, but this is a mere *semblance*, *relative* to the basis (except, of course, if the corresponding measurement is performed, which will *produce* the mixture).

V) *Concluding*, if the recent measurements (1) had been performed before the advent of the new quantum mechanics, they certainly would have produced the same stupefaction as the Michelson experiment did. Einstein (2) did point out at the paradox as early as 1927, but the simple calculations of this paper were not produced at that time.

That extremely simple calculations do best convey the new *paradigm* implied in a true, or real, paradox (8), has been exemplified by both Einstein (for special relativity) and de Broglie (for wave mechanics).

The present situation resembles much the one out of which special relativity was born: we do have the good formulae, but have not yet fully understood their meaning. And today, like then, the answer to the riddle is not to be found inside an underlying *modelism* (« hidden variables » replacing the « mechanical ether »), but rather in an unbiased reading of both the experimental result and the operational *formalism*.

What the experiments do say is that the measurements performed at the distant places L and N do produce the *same* wave collapse at C, *in their common past*. And what the (relativistically covariant) formalism does say (9) is that the neoquantal stochastic event, the *wave collapse*, is (as was the one in classical statistical mechanics) *intrinsically time symmetric*, affecting both future and past.

Thus, Einstein's prohibition to telegraph into the past was only a *macroscopic* or *de facto*, one. It does not hold at the elementary level. This, together with the golden rule of the *wavelike probability calculus* (addition of partial *amplitudes*) requires (no less than did the Michelson experiment) a drastic revision of « our classical ideas pertaining to space and time » (6).