IS THERE A PARADOX IN THE THEORY OF TIME ANISOTROPY?

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Before we formulate (in accord with quite a few recent authors) the answer appropriate to the question in the title, it is useful to understand why it has been felt that the theory of the time anisotropy contains a paradox.

I. THE PARADOX BEHIND THE LOSCHMIDT AND ZERMELO PARADOXES: TIME ANISOTROPY IN THE "PRINCIPLE OF PROBABILITY OF CAUSES"

The well known Loschmidt and Zermelo "paradoxes" in statistical mechanics have merely uncovered the existence of a much older "paradox" inherent in the probability theory itself since the early days of Pascal, Fermat, and Bayes, where it came to be named, very significantly, the "principle of probability of *causes*." That is to say, it was obscurely felt that the time anisotropy inherent in the anthropomorphic notion of a "cause" developing after effects rather than before effects is somehow connected with the empirical fact that, *even if the transition probabilities between two possible states of a system are symmetric* (as in such classical examples as card shuffling or dice throwing), more-probable macroscopic complexions follow in time the less-probable ones—not the other way around.

As Watanabe puts it, the empirical fact is that blind statistical prediction is physical while blind statistical retrodiction is not—a situation with which probability theory copes by using in retrodictive problems Bayes' formula for conditional probability. But, as the Bayes coefficients are by definition independent of the internal dynamics of the system under study, this amounts to saying that the theoretical description of time anisotropy in probability problems is of an extrinsic rather than intrinsic nature. It is, very exactly, a *boundary condition* imposed upon the macroscopic evolution equations. This boundary condition reads "blind retrodiction forbidden"—very much as the boundary condition in macroscopic wave theories reads "advanced waves forbidden." The point is that in both cases the boundary condition is an initial, not a final condition. To this we will come back later.

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Now I stress the connection between the temporal application of Bayes' principle and the causality concept. To say that the evaluation of Bayes' coefficients is extrinsic to the dynamics of the system is to say that they are used to describe an interaction between the system and its surroundings. And to say that the Bayes coefficients must be used in retrodictive but not in predictive problems amounts to saying that the effects of the interaction upon the system are felt after it has ceased, and not before it has begun. But this is the very definition of causality—that is, "retarded actions," as opposed to finality or "advanced actions."

A typical example of this general physical law is the ink drop that dissolves in a glass of water after it has been deposited in it by a pipette; the reversed procedure including, of course, the pipette and the man holding it, can only be seen by viewing a movie film reversed. The same can be said of card shuffling.[1]

We thus come to the important conclusion that the time dis-symmetry inherent in causality as opposed to finality is of an essentially macroscopic nature, and that its mathematical formulation consists in the temporal application of Bayes' principle expressing that blind statistical retrodiction is forbidden. To my cognizance Van der Waals was the first (1911) to state that the statistical derivation of Carnot's law is merely an application of Bayes' principle—an idea which is also implicit in an often quoted sentence of Willard Gibbs (1914).[2] That the mathematical expression of statistical irreversibility is, as Mehlberg (1961) puts it, of a fact-like rather than law-like character has also been expressed in recent years by Watanabe, Reichenbach, Adams, McLennan, Wu-Rivier, Grünbaum, von Weiszäcker, Ludwig, and Costa de Beauregard. The identification of the causality concept with the physical law of increasing probabilities has been especially strongly stressed by Reichenbach, Grünbaum, Terletsky, and Costa de Beauregard.

II. THE EINSTEIN-RITZ CONTROVERSY: RETARDED WAVES AND PROBABILITY INCREASE

The existence of a close connection between the two principles of wave retardation and probability increase is strongly suggested by many physical

[1] Of course a deck of cards is said to be "in order" or "in disorder" according to the fact that their sequence does or does not belong to some selected small sub-ensemble of the ensemble of possible permutations of the cards.

[2] "It should not be forgotten, when our ensembles are chosen to illustrate the probabilities of events in the real world, that while the probabilities of subsequent events may often be determined from those of prior events, it is rarely the case that probabilities of prior events can be determined from those of subsequent events, for we are rarely justified in excluding the consideration of the anteceding probability of the prior events."

examples such as, for instance, the slowing down of a meteorite in the earth's atmosphere. In these recent years it has been more or less explicitly stated, in various contexts, by quite a few authors among those listed at the end of this paper (McLennan, Penrose-Percival, Costa de Beauregard, and others). To my cognizance the discussion started in the 1900's with the celebrated Einstein-Ritz controversy, in which Ritz insisted on deducing the law of entropy increase from the principle of wave retardation while Einstein maintained that the law of wave retardation should follow from the principle of probability increase.

That the Einstein and Ritz statements are *reciprocal* should be obvious now that the formulation of the principle of statistical irreversibility has been recognized to be of the nature of a boundary condition.

The aim of this Section is to show that, in a restricted but precise context, wave retardation and probability increase are indeed two names for one and the same principle. To this end we will work with a theory implying *essentially* the two concepts of waves and probability, namely, quantum mechanics.[3]

Let me first illustrate my statement by using an example. A plane monochromatic wave falling upon a linear grating (the wave planes being parallel to the lines on the grating, which we assume infinite in number for simplicity) generates a finite number g of outgoing plane monochromatic waves; this follows from the necessity of phase coherence and the principle of retarded waves. Now, if one of these outgoing plane waves is received in a collimator and the observer knows nothing other than the presence of the grating (and, of course, the space frequencies of both the wave and the grating), the only thing he can say is that an incoming plane wave falls on the grating, and that it is one among a well defined class of g waves (comprising the one considered first). He does *not* conclude that the wave he receives is built up by phase coherence of the g possible incident waves, because this would imply acceptance of the macroscopic existence of advanced-waves.

Now we must remember that our waves are assumed to be quantized, so that we can transpose a discourse on intensities into a discourse on probabilities. If, in a predictive problem, n corpuscles with the same sharply defined momentum fall per unit time upon the grating, then, to use Watanabe's excellent terminology, a "blind statistical prediction" yields

[3] The connection between the two principles of probability increase and wave retardation is implied in Planck's definition of the entropy of a light beam, where the constant h—that is, the photon concept—is essential.

It follows from Planck's formula that the entropy of a light beam is increased by scattering or "*dif*-fusion", a time asymmetric process following from the principle of retarded waves. If advanced waves were macroscopically existent, then phase-coherent "*in*-fusion" would decrease the entropy of the light beam.

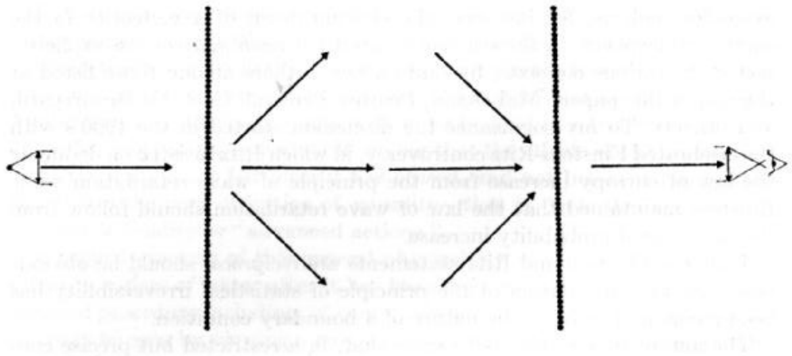

Fig. 1. Physical irreversibility as a boundary condition: the quantized wave and grating thought experiment. a) Retarded waves and blind statistical prediction. b) Advanced waves and blind statistical retrodiction.

$n!/\pi(n_i!)$ as the probability $P(n_i)$ that n_i corpuscles per unit time come out on each of the admissible outgoing waves; for simplicity the transition probabilities between the g initial and the g final mutually exclusive states are assumed to be equal; also, n is taken low enough for each wave train to carry not more than one corpuscle, so that we can neglect quantum statistical interactions. The above probability $P(n_i)$ is maximized when all the n_i's are equal; this is the statistical transposition of the classical computation of intensities using the principle of retarded waves. The point is that, in the "retrodictive problem," where n corpuscles per unit time are received on one outgoing plane wave, a "blind statistical retrodiction" would entail the paradoxical conclusion that the incoming particles were equally distributed among the g admissible incoming waves. But, according to the now accepted view, the principle of statistical irreversibility in physics amounts to the boundary condition that *blind statistical retrodiction is forbidden*. We thus come to the conclusion that, in the theory of quantized waves, the principle that blind statistical retrodiction is forbidden is just another wording for the principle that advanced waves are *macroscopically* nonexistent.

The consideration of this (or any similar) example is a good preparation for understanding the abstract proof of the equivalence in quantum mechanics of the two irreversibility principles that will now be presented. This proof merely consists of a rewording of von Neumann's celebrated theorem on statistical irreversibility in the measuring process.

Denoting $| i\rangle p_i \langle i |$ as the density matrix and $| j\rangle\langle j |$ the projector describing some pure state $| j\rangle$, the statistical frequency p_j' of the state $| j\rangle$ at the end of the measurement is (no summation on j!)

$$p_j' = \text{Trace} \,| i\rangle p_i \langle i | j\rangle\langle j | = \sum_i p_i \,| \langle i | j\rangle |^2.$$

Thus, denoting P as the largest p_i and using the normalization condition,

$$\sum_i | \langle i | j\rangle |^2 = 1,$$

the result

$$p_j' \leq P \quad \text{for all} \quad j\text{'s}$$

follows; this is a typical instance of the levelling of statistical frequencies in blind prediction—that is, physical prediction.

The point is that the principle of retarded waves has been implicitly used when stating that, *macroscopically speaking in the sense of von Neumann's ensembles*, $| \langle i | j\rangle |^2 < 1$ is the predictive and *not the retrodictive* probability of the transition $i \to j$.

In other words, with the use of retarded and advanced waves, respectively, in quantum theory, for statistical prediction and retrodiction, to say that advanced waves are *macroscopically* nonexistent or that blind statistical retrodiction is forbidden are two different wordings for one and the same statement. Both ways, macroscopic irreversibility is extracted from microscopic time symmetry *via* a boundary condition—very much as one-way driving is secured by the appropriate sign post.

Finally, I must emphasize that the clarity of the connection we have established between wave retardation and probability increase is such by virtue of a precise, but narrow context. Von Neumann's micro-entropy (which, for brevity, has not been introduced explicitly) is a much simpler concept than the macro-entropy of thermodynamics. Nevertheless, as I have said, there are so many examples of an observable connection between wave retardation and entropy increase that I am confident that this kind of argument can be very largely extended. For instance, if between time instants t_1 and t_2 a physicist moves a piston in the wall of a vessel containing a gas in equilibrium, the fact is that Maxwell's velocity distribution law is altered after time t_2, not before time t_1. But, the fact is also that the perturbation is emitted (not absorbed) by the moving piston as a retarded (not advanced) wave.

III. THE FACT-LIKE CHARACTER OF PHYSICAL IRREVERSIBILITY, AND THE INFORMATION CONCEPT

It is certainly very striking that cybernetics has rediscovered, without having searched for it, the twofold aspect of the old Aristotelian "information" concept—namely, (i) gain in knowledge and (ii) organizing power. That (except in a few philosophical circles interested in finality) the second aspect of Aristotle's "information" happened to be almost completely forgotten can be understood as a consequence of physical irreversibility, as is now explained.

That information is a two-faced concept in cybernetics is quite obvious in the characteristic chain

$$\textit{information}_1 \rightarrow \textit{negentropy} \rightarrow \textit{information}_2$$

of communication systems or computers; also, in the characteristic chain

$$\textit{negentropy}_1 \rightarrow \textit{information} \rightarrow \textit{negentropy}_2$$

of physical measurements and the classifications they allow such as, for example, in the Maxwell demon problem analyzed by Smoluchowsky, Szilard, Demers, Brillouin, or the von Neumann measuring process in quantum mechanics The *learning transition*

$$\textit{negentropy} \rightarrow \textit{information} \qquad \text{(i)}$$

appears there as symmetrical to the *acting transition*

$$\textit{information} \rightarrow \textit{negentropy}. \qquad \text{(ii)}$$

In (i), *observational awareness* follows in time the physical situation which, in accordance with Reichenbach's analyses, it *registers*. In (ii), *willing awareness* precedes in time the physical situation which it contributes to *produce*.

Physical irreversibility consists of the *fact* that the above arrows all point toward lower information or negentropy values. Thus, for instance, the learning transition (i) appears as a generalization of the passive Carnot degradation of negentropy in closed systems. But, as Mehlberg and others have so strongly stressed, physical irreversibility is of a fact-like rather than a law-like character. Thus, cybernetics implies an invitation to inquire about the law-like rather than the fact-like status of our problem—very much like the internal symmetries of the Dirac electron theory that have been justified by the discovery of Anderson's positron which, though *de facto* much rarer than the electron, is *de jure* its twin brother.

The irreversibility principle, as stated in the two preceding sections, amounts to saying that, physically speaking, a low probability complexion can be taken as the starting point of a regressing fluctuation rather than the end point of a progressing fluctuation. Nevertheless, it is well known that, even in physics, progressing fluctuations *do* exist. It is thus clear that the categorical character of the physical irreversibility statements (as previously expressed) stems from the implicit assumption that very large

ensembles are at stake; more-generally speaking, these statements should be understood in a probabilistic (a *predictive probabilistic*) sense.

So, the fact-like (*not law-like*) irreversibility principle of cybernetics turns out to be that the learning transitions (i) are more probable than the acting transitions (ii). In terms of awareness, observation is easier, or less tiring, than action.

Now, it is quite clear that the very value of universal constants in terms of "practical," or anthropomorphic physical units, reflects an existential situation. For instance, to say that the value of the light velocity c is "very large" is to say that the ratio of associated length and time units we find convenient is much smaller; this fortuitous circumstance (which, in my opinion, may well stem from the very value of our nervous influx velocity: a small multiple of 1 cm $\sec^{-1}$) is, as is now well known, at the origin of everybody's feeling that there is an absolute time. Quite similarly, the "smallness" of Boltzmann's constant k and of the conversion coefficient $k \ln 2$ between an entropy expressed in "practical" thermodynamic units and an information expressed in binary units according to the equivalence formula

$$\textit{negentropy} = k \ln 2 \times \textit{information}$$

may well be taken as a direct expression of the *fact* that observation is for us much easier than action: the conversion rate is such that gaining knowledge is very cheap in negentropy units, while producing negentropy costs a lot in *bits*. Going to the limit $k \rightarrow 0$ would imply that observation is completely costless, and action impossible. This crude approximation to cybernetics has been known as the theory of "epiphenomenal consciousness."

My final remark will be nearly philosophic. For a long time, it has been recognized that progressing fluctuations and advanced waves can be taken as the objective aspect of finality, just as regressing fluctuations and retarded waves are now understood to be *the* physical expression of causality. There should then be no wonder that the finality concept is so elusive in terms of cognitive awareness, for it follows from the above that the learning transition being "causal" and the acting transition being "final" in the above sense, the evidence of causality belongs to cognitive awareness just as the evidence of finality belongs to willing awareness. This, of course, is well known to philosophers, but cybernetics helps understanding why things are so.

CONCLUSION

We have resumed in modern terms the old "paradoxical" problem of deducing physical irreversibility from elementary laws assumed to be time symmetric. We have found, with quite a few recent writers, that strictly speaking there is no paradox at all, but merely a *fact-like* state of affairs which is mathematically expressible as an appropriate boundary condition.

As for the law-like status existing beyond the fact-like situation, I feel that cybernetics has something to say. Very strikingly, a quantum measurement essentially implies a perturbation on the measured system so that, in this sense, cognizance and action are inseparable. More thinking and more knowledge are needed here, and I feel that quantum biology might well take part in the discussion.

Finally, there is of course the possibility that the recent *PC* violations in elementary particle physics imply *T* violations that should be superimposed as a slight perturbation on the preceding scheme. But this is part of tomorrow's problems.

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Comments

T. E. Phipps: I think, relevant to the question of irreversibility that you have discussed, perhaps a quotation from Seussmann in the *Observation and Interpretation* book that came out in about 1957 is worth mentioning.

He said, "Besides physics, there is history." Think about it. This remark is actually a critical statement against the existing quantum mechanical theory of measurement, the implication being that there are things we know retrospectively, such as specific locations of scintillation points on phosphors, grain darkenings on photographic plates, and so on, which the formalism lacks parameters to describe; that is, the formalism as it exists now. The accepted quantum equations of motion, taken alone, describe the class of all physically admissible histories, but not the specific history that was observed. This is perhaps germane to your mention of the fact-like situation.

The trouble is that the formalism, if we restrict our attention to equations of motion alone, contains no parameters to describe the fact-like situation retrodictively. (Hence, the theory can accord also predictively no meaning to the fact-like situation.) One therefore seeks to supplement the formalism, and von Neumann's projection postulate was an effort to do this. It was an effort that was very severely criticized by Henry Margenau as early as 1937. To my mind, the postulate never really survived the critical onslaught that Margenau made on it, although it is still printed in many books and papers on measurement theory. So the gist of my remark is that I think we still have work to do in the quantum mechanical theory of measurement, bearing on the whole question of retrodiction and irreversibility.

O. Costa de Beauregard: Yes, I think I agree with what you said. You see, the gist in my argumentation is the symmetrical consideration of prediction and retrodiction. I believe *this* is the problem behind the Loschmidt and Zermelo paradoxes, and I am convinced that the root of physical irreversibility is not in the laws but in the boundary conditions; and this is a conclusion that has been reached by quite a few authors in these recent years.

T. E. Phipps: Yes, I am agreeing with that, but I am saying also that one of the formalistic problems is that the accepted formalism contains no parameters to describe the boundary conditions you mention. That in consequence of this parameter deficiency only the class of all physically admissible situations is described by accepted quantum equations of motion (including time reversed situations); hence that such equations do not suffice to describe either the specificity or the irreversibility of experience.

P. T. Landsberg: I don't know whether what I am going to say is at all sensible, but it may be suitable for the panel meeting later. Supposing that we were within an ink drop, and able to see all the processes going on in great detail by being a tiny microbe. The whole thing would be chaos to us. We would see particles going forward and backward, and everything would seem to be reversible. But we are actually not able to follow these microscopic processes in all detail. After all, the eye is a very rough instrument. So we are averaging over regions. We don't have all the information we could in principle have and, therefore, I would ask you what do you think: May it be that the irreversibility is due to what I might call the "macroscopic nature of time" that is the impossibility of really seeing the detailed processes.

If we could see the detailed processes we would be in chaos, and everything would be reversible. Since we cannot, maybe the origin of irreversibility might be found in that area. I am asking; I don't know.

O. Costa de Beauregard: Yes, I agree much with what you are saying, but, of course, as you know still better than I do, it is a very subtle problem.