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L. Kubat and J. Zeman eds

CAN INFORMATION THEORY RECEIVE A RELATIVISTIC EXPRESSION?

O. Costa de Beauregard

Bergson recounts somewhere that, having once been asked what to-morrow's novel would look like, he answered that he did not know, for, had he known, this would imply that he had written the novel.

Well, up to now, only this title of the novel exists. Notwithstanding Bergson's serious warning let us try to outline what it should like.

Speaking of *relativity theory* implies at once that the time concept is taken in a strongly physical sense, and not in some remotely abstract fashion as may be the case in an academic version of probability theory or information theory. We will be dealing with some manifestly covariant form of statistical mechanics.

It so happens that Jaynes [1] partly inspired, as it seems [2], by Cox [3], produced a self-contained general formalism of statistical mechanics, yielding an extremely concise and transparent deduction of all the essentials. Later Tribus [4] published elegant presentations of this doctrine and Katz [5] has developed it in the form of a treatise.

Jaynes' prescription for obtaining the significant probability distribution is derived straight from information theory and reads: Maximize the entropy

$$S \equiv -\Sigma p_i \ln p_i \tag{1}$$

subject to

$$\Sigma p_i = 1 \tag{2}$$

and whatever contraints result from the physical situation. For instance, in classical statistical mechanics, the mean value $\langle E \rangle$ of the energy E, and the mean values $\langle n_m \rangle$ of the numbers of different chemical molecules m, are given, whence

$$\Sigma p_i E_i = \langle E_i \rangle, \tag{3}$$

$$\Sigma p_i n_{mi} = \langle n_m \rangle \tag{4}$$

etc... Then the well known technique of Langrange multipliers yields, through the definition of the partition function

$$\pi = \ln \sum_i e^{-\beta E_i - \sum_m \beta_m n_{mi}}, \tag{5}$$

the following expressions for the entropy, the mean energy, the mean concentrations, etc...

$$S = \pi + \beta \langle E \rangle + \sum_m \beta_m \langle n_m \rangle, \tag{6}$$

$$\langle E \rangle = -\partial\pi/\partial\beta, \tag{7}$$

$$\langle n_m \rangle = -\partial\pi/\partial\beta_m. \tag{8}$$

Of course, the prescription and the formulas are nothing more than what is found in every classical exposition of equilibrium statistical mechanics. What is new is that the line of reasoning is reduced to its bare skeleton, plus the motive muscles enabling it to be applied directly for solving any problem within its potential field. The huge obesity of classical statistical mechanics—that is, the extensive considerations on ensembles and so on—is completely gone, much obscurity in applications or extentions has also disappeared, and much clarity has been gained.

As for relativistic covariance there is *a priori* "no problem". The exponent in formula (5) must be a scalar. Therefore, if we insert instead of the energy E the energy-momentum 4-vector E^λ, with $\lambda = 1, 2, 3, 4$, the "inverse temperature" β inside the scalar product $\beta_\lambda E^\lambda$ will show up as a 4-vector β_λ. If we need to introduce the six-component angular momentum $E^{[\lambda\mu]}$, a corresponding skew-symmetric inverse temperature $\beta_{[\lambda\mu]}$ will come in. As for the β_m's, they will of course remain scalars, etc...

Therefore, the relativistic covariance should not bring in *per se* any serious problem.

In a physically significant information theory, we also need a covariant formalism for information transfer. At this end wave propagation is ideally suited. Moreover, as we should not be restricted to electromagnetic waves, it is only natural to turn towards wave mechanics, that is, quantum mechanics.

So the question naturally arises: can we find, in the quantum mechanical formalism, an acceptable definition of entropy "corresponding" to the classical one? The answer is known to be *yes* – through J. von Neumann's definition [6] of the "density matrix" P and his "ensemble" presentation of quantum mechanics. The entropy is then defined as

$$S = -\text{Trace}\,(P \ln P). \tag{9}$$

It so happens that in an important, but rarely quoted article of 1937, Elsasser [7] inspired by Fisher [8] has applied, inside the von Neumann scheme, the very prescription later used by Jaynes, in order to produce a consistent theory of the quantum mechanical measuring process. In Elsasser's words [7, p. 992] this was an information theory approach. He also states that, in his line of thought, the density matrix should be used for describing not only ensembles, but even individual systems.

So one more of the things we are needing is already there: an anticipated sketch of the use of Jaynes' formalism inside the scheme of J. von Neumann's quantum mechanical density matrices *plus* an explicit application of it by Jaynes himself.

The next thing we need is a relativistically covariant formalism for the scalar product, the normalization and orthogonality formulas, the reciprocal Fourier transforms, all of which are the everyday bread of quantum mechanics, and thus the basis for constructing, in any specific case, the von Neumann density matrix.

Of course, since the celebrated works of Tomonaga [9], Schwinger [10], Feynman [11] and Dyson [12], everybody knows that a beautiful formalism exists for relativistic quantum electrodynamics, and some other specifications of quantum field theory. What we need for our purpose is something more down to earth: it is a relativistically covariant presentation of the "first" rather than of the "second quantized" mechanics.

It so happens that this very problem has been with me for many years, and that, after writing a few articles on it [13], I have expanded the whole subject in a booklet [14]. Wightman and Schweber [15] have also considered the problem.

So what seems to be the last necessary piece of our puzzle is also there: an explicitly covariant formulation of "first quantization", including a theory of Green's functions or "propagators", and a resolution of the Cauchy problem in the form of the expansion of a wave function on a complete set of orthogonal propagators.

So then what remains to be done? At this point one should remember Bergson's warning: building up an operational physical theory is not rattling on or waving hands. That is, building up a theory is *never* merely putting together pieces of a puzzle; one can never avoid using sand-paper – to say the least. Therefore it is *only* when building up the whole scheme that one will discover what it truly looks like – and almost certainly one will discover unexpected implications.

But man's mind is thus made that he cannot help trying to guess what unexpected things he should expect. In my opinion, there is one information transfer problem in space-time that is a real big one: the one pertaining to the so-called Einstein-Podolsky-Rosen [16] paradox, also considered by Schrödinger [17] and by Furry [18]. As I have already expressed [19] my feelings in this respect I will not do it again here. I will simply draw attention to a very interesting recent paper by Moldauer [20] entitled "Is there a quantum mechanical measurement problem?", the implication being that there is none. The case is made very strongly, and formulas for conditional probability are given. Therefore the paper is important to us.

However, notwithstanding Moldauer's dazzling exorcism of ghosts, I believe that the problem of information gain (knowledge), of information use (action), and of information transfer by means of waves still contains, in quantum mechanics, an unsolved mystery.

The point is clearly made by Hooker [21] and, in my opinion, brings us straight back to London and Bauer's [22] philosophy. In the Einstein-Podolsky-Rosen "paradox" the problem is neither prediction, nor even retrodiction, but truly *telediction*, along a spacelike separation built up from two timelike separations making a sort of Feynman zigzag [19]. It is *not* relevant for observer A to think of the other distant observer B as going to perform, or having already performed, his measurement when A is performing his own one, because this order in time is a *relative* and not an *absolute* one.

The emphasis here is on time symmetry of elementary processes. This calls for full consideration of the *intrinsic* symmetries between statistical prediction and retrodiction, retarded and advanced waves, information as knowledge and as volition, as discussed by quite a few authors [23].

To conclude, I do hope that a student of mine will show up and be ready to tackle these intertwined problems, because I certainly would like to look over his shoulder while the work is going on.

Also, I should like to say that the names I have cited in my paper are certainly not the only ones that could have been quoted. The approach to probability problems and to statistical mechanics, either classical or quantal, has truly been, either explicitly or implicitly, "in the air" for all these past years.

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