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# PHYSICAL REALITY OF ELECTROMAGNETIC POTENTIALS ?

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Although the force $c^{-2}$ (d$i$/d$t$) $\oint V\delta \boldsymbol{l}$ we have shown [1] to be exerted on a varying current by a constant electric field $\boldsymbol{E} \equiv$ grad $V$ is gauge independent, it would be hard to deny the existence of the local force $c^{-2}$ (d$i$/d$t$) $V\delta \boldsymbol{l}$.

We have proved recently [1] that a closed circuit C carrying a current of variable intensity $i(t)$ undergoes, in a time independent external electric field $\boldsymbol{E} \equiv$ grad $V$, a force

$$\boldsymbol{F} = c^{-2}(\mathrm{d}i/\mathrm{d}t) \iint_S \boldsymbol{E}\times\delta \boldsymbol{s} \equiv c^{-2}(\mathrm{d}i/\mathrm{d}t)\oint_C V\delta \boldsymbol{l}\,(\mathrm{e.m.u}) \qquad (1)$$

the first expression of which is in terms of the equivalent magnetic layer. An alternative proof is as follows. According to the well-known Ampère formula $i = \oint \boldsymbol{H}\cdot \mathrm{d}\boldsymbol{l}$ where the integral is along an arbitrary closed contour circling C once, and due to the fact that the $\boldsymbol{H}$ and $\delta \boldsymbol{s}$ vectors are collinear when S is varied inside the class of the equipotentials of the magnetic layer, formula (1) may be rewritten as

$$\boldsymbol{F} = c^{-2} \iiint \boldsymbol{E}\times \partial \boldsymbol{H}/\partial t\, \mathrm{d}v \qquad (2)$$

where the integral is over all space with $\mathrm{d}v \equiv \delta \boldsymbol{s}\cdot \mathrm{d}\boldsymbol{l}$. The force $\boldsymbol{F}$ is thus expressed as the time derivative of the integrated Poynting vector - a quite standard explanation.

The point we intend to stress is that, although formula (1) is gauge independent, it would be hard to understand it if it were denied that formula

$$\delta \boldsymbol{F} = c^{-2}\,(\mathrm{d}i/\mathrm{d}t)\, V\, \delta \boldsymbol{l} \qquad (3)$$

has local validity. Consider for instance (fig. 1) a circuit C consisting of two straight parallel branches L of common length $L$ connected by two small semi- circles R, the two L branches being inside the cylindrical plates, of equal diameters $a$ and length $L$, of a charged capacitor. For given charges $+q$ and $-q$ per unit length of the capacitor and a given d$i$/d$t$ in the circuit, the force $\boldsymbol{F}$ of formula (1) (precisely opposite to the force $-\boldsymbol{F}$ exerted on the capacitor) is here proportional to $L$. As the electric field $\boldsymbol{E}$ (which is non-zero

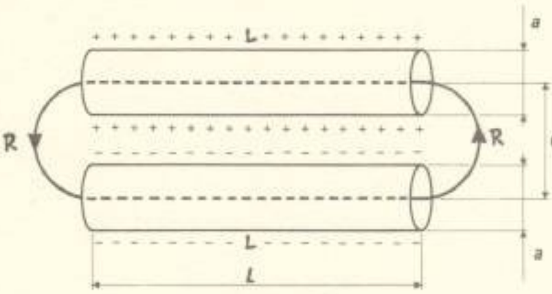


Fig. 1. Conservation of linear momentum : variable current and charged capacitor ($a \ll L$, $d \ll L$).

only outside the cylinders) is, on the R sections of C, $L$ independent, the latter result can hardly be understood if it is denied that the gradientless potential inside the cylinders exerts the elementary force (3) [2]. Then, the principle of conservation of linear momentum selects the Coulombian gauge of $V$ for, if $V$ is generated by a point charge $Q$ and has the Coulomb value $(4\pi r)^{-1}Q$, the elementary force exerted on $Q$ by the induced electric field is precisely $-\delta \boldsymbol{F}$.

The foregoing argument belongs to the same general family as de Broglie's [3] which, using the relativistic equivalence of energy and mass and the expression of the electromagnetic mass defect in the action-at-a-distance formulation of electromagnetism, selects the Liénard-Wiechert gauge. Also, the (gauge independent) Aharonov-Bohm [4] and Mercereau [5] effects have in this line a kinship with the present one, because in them the significant magnitude is the potential momentum $Q/A$.

Obviously, the essential difference between the classical version of electromagnetism and the one we tentatively present here consists in the nonexistence or existence of the *local* force

[3]. Unfortunately, due to the presence of the $c^{-2}$ factor in eq. (3), the crucial experimental test is practically not feasible in this form *.

1. O. Costa de Beauregard, Physics Letters 24A (1967) 177. It is unfortunate that a factor $c^{-2}$ (in coherent units) has been dropped before the expressions of the force $F$.
2. Explicit formulas are given in O. Costa de Beauregard, Compt. Rend. 263B (1966) 1047.
3. L. de Broglie, Compt. Rend. 225 (1947) 163.
4. Y. Aharonov and D. Bohm, Phys. Rev. 115 (1959) 485; W. Ehrenberg and R. E. Siday, Proc. Phys. Soc. B62 (1949) 21.
5. R. C. Jaklevic, J. J. Lambe, A. M. Silver and J. E. Mercereau, Phys. Rev. Letters 12 (1964) 274.

* Conservation laws associated with the torque $2c^{-2}V\,d\boldsymbol{M}/dt$ which, according to the present theory, is applied to an Amperian current or dipole of moment $\boldsymbol{M}$ by an external potential $V$ will be discussed in an other paper.

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