PHYSICAL NON-REALITY OF ELECTROMAGNETIC POTENTIALS

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In answer to Golub’s [1] recent experiment, a new Gedankenexperiment is devised showing the dilemma between the equality of action and reaction and equivalence of Coulomb’s and Ampère’s pictures of magnetism and the existence of a force $c^{-2} \frac{1}{\varphi} \frac{\partial \varphi}{\partial t}$ applied to a closed current by a variable electric potential.

Golub et al. [1] have recently reported an experiment which may be taken as indirect proof that the local force $c^{-2} \frac{\partial \varphi}{\partial t} \vec{V} \delta \vec{r}$ does not exist, the integral, gauge independent, expression $c^{-2} \frac{\partial \varphi}{\partial t} \oint \vec{V} \delta \vec{r}$ is that of a force applied to the total current; but, as was explained [2] this would be hard to understand.

II. The integrated force does not exist either; then two unavoidable consequences would be (A) the non equality of action and reaction applied to the sources of the field [2] and (B) the non equivalence of the Coulomb and Ampère pictures of magnetism [3].

We thus feel that the problem we have raised [2, 3] is a fundamental one, and will present here a new Gedankenexperiment in support of this claim.

A rectilinear wire of finite length is slowly charged according to the law

$$j = Kt, \quad q = Kt,$$

where $K$ denotes a constant, $t$ the time, $j(\varphi, t)$ the current intensity and $q(\varphi, t)$ the charge per unit length. Such a situation can last a sufficiently long time to display the paradox we have in view.

A closed toroidal solenoid of axis $z'$ carries a time independent current $i \delta \varphi$ per longitude angle $\delta \varphi$ (natural cylindrical coordinates $p, \vartheta, z$ are used), and thus a "total current* $I = 2 \pi i$. As the magnetic field $H_1 = \text{curl} \vec{A}_1$ of the rectilinear wire is time independent no extra current is induced in the solenoid*.

Thus, the rectilinear wire $z'$ and/or the charges therein feel no force due to the presence of the $I$ current in the solenoid. The paradox is that the solenoid feels an overall Laplace force of direction $z$ and value

$$F_{1z} = I \frac{\partial \varphi}{\partial z}$$

with $\phi$ denoting the flux of $H_1$ through a semimeridian section $S$ of contour $C$

$$F_{1z} = I \oint \vec{g} \cdot (\partial H_1 / \partial z), \delta s = I \oint \vec{g} \cdot (\partial A_1 / \partial z), \delta z$$

The paradox reaches a climax when we remark that the radiation expression of the reaction to the force [2], i.e., $H_2$ denoting the solenoid’s magnetic field,

$$F_{2z} = c^{-2} \oint \oint j(\partial E_1 / \partial t) \times H_2, \delta \varphi$$

with the integral taken inside the solenoid this is why our Gedankenexperiment implies a closed toroidal solenoid: to have $H_2$ trapped in its interior. Substituting in (4) $H_2 = I / 2 \pi p$ yields (as $E_1 = \text{grad} V_1$, due to the fact that $\partial A_1 / \partial t = 0$)

$$F_{2z} = c^{-2} \oint \oint j(\partial E_1 / \partial t) \delta \varphi =$$

$$= c^{-2} \oint \oint (\partial V_1 / \partial t) \delta \varphi.$$

Substituting in eqs. (4) and (5)
\[ dA_x = (jdz/4\pi \gamma) \hat{n}, \quad dV_1 = e^2 jdz/4\pi \gamma, \]  
\[ \text{with} \ \hat{n} \text{denoting the unit vector of the z axis and} \ \gamma \text{the distance from the dz element}, \]  
\[ \text{yields as was announced,} \]  
\[ d(F_1 e + P_2 ez) = 0. \]  

Finally the dilemma is:
II. The force [5] does not exist, in which case there is neither equality of action and reaction nor equivalence of the Coulomb [4] and Ampère pictures of magnetism.

Incidentally, the problem of existence or non-existence of the force \( e^{-2} \frac{d}{dt} \mathbf{F} \) of \( \mathbf{V}_1 \) by can be cast in the form of a Gedankenexperiment parallelising this one.

References
1. L. Golub, J.P. Carrie, T.S. Stein and M.C. Wesskopf, Phys. Letters 25A (1967) 595. One thing that makes the proof questionable is that in our paper [2] the magnetic moment change follows the intensity change in a fixed circuit, while this is obviously not the case in Golub’s experiment.

** For given \( r \) and \( \Delta t \equiv \pi e \), \( dA \) and \( dV_1 \) and \( dV_1 \) satisfy the Lorentz condition.