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PHYSICAL NON-REALITY OF ELECTROMAGNETIC POTENTIALS?

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In answer to Golub's [1] recent experiment, a new Gedankenexperiment is devised showing the dilemma between the equality of action and reaction and equivalence of Coulomb's and Ampère's pictures of magnetism and the existence of a force $c^{-2}\, i \oint (\partial V/\partial t)\,\delta l$ applied to a closed current by a variable electric potential.

Golub et al. [1] have recently reported an experiment which may be taken as indirect proof that the *local force* [2] $c^{-2}\, \mathrm{d}i/\mathrm{d}t\; V \delta l$ e. m. u. applied to a variable current in a fixed wire by an external electric potential has no physical existence. Two possibilities are then left:

I. Although the above local force does not exist, the integral, gauge independent, expression [3] $c^{-2}\, \mathrm{d}i/\mathrm{d}t \oint V \delta l$ is that of a force applied to the total current; but, as was explained [2] this would be hard to understand.

II. The integrated force does not exist either; then two unavoidable consequences would be (A) the non equality of action and reaction applied to the sources of the field [2] and (B) the non equivalence of the Coulomb and Ampère pictures of magnetism [3].

We thus feel that the problem we have raised [2, 3] is a fundamental one, and will present here a new Gedankenexperiment in support of this claim.

A rectilinear wire of finite length is slowly charged according to the law

$$j = -Kz, \quad q = -Kt, \tag{1}$$

where K denotes a constant, t the time, $j(z,t)$ the current intensity and $q(z,t)$ the charge per unit length. Such a situation can last a sufficiently long time to display the paradox we have in view.

* Of course, the time variation of the q distribution on $z'z$ induces static charges in the solenoid. But the corresponding law being I independent, this is an independent energy-momentum balance problem.

A closed toroidal solenoid of axis $z'z$ carries a time independent current $i\,\delta\theta$ per longitude angle $\delta\theta$ (natural cylindrical coordinates ρ, θ, z are used), and thus a "total current" $I = 2\pi i$. As the magnetic field $H_1 = \mathrm{curl}\, A_1$ of the rectilinear wire is time independent no extra current is induced in the solenoid *.

Thus, the rectilinear wire $z'z$ and/or the charges therein feel no force due to the presence of the I current in the solenoid. The paradox is that the solenoid feels an overall Laplace force of direction z and value

$$F_{1_z} = I\, \partial\Phi/\partial z \tag{2}$$

with Φ denoting the flux of H_1 through a semi-meridian section S of contour C:

$$F_{1z} = I \iint_S (\partial H_1/\partial z)\,.\,\delta s \equiv I \oint_L (\partial A_1/\partial z)\,.\,\delta l\,. \tag{3}$$

The paradox reaches a climax when we remark that the radiation expression of the reaction to the force [2] is, H_2 denoting the solenoid's magnetic field,

$$F_{2_z} \equiv -F_{1_z} = c^{-2} \iiint_v (\partial E_1/\partial t) \times H_2\, \delta v \tag{4}$$

with the integral taken inside the solenoid this is why our Gedankenexperiment implies a closed toroidal solenoid: to have H_2 trapped in its interior. Substituting in (4) $H_2 = I/2\pi\rho$ yields (as $E_1 = \mathrm{grad}\, V_1$, due to the fact that $\partial A_1/\partial t = 0$)

$$F_{2_z} = c^{-2}\, I \iint_S (\partial E_1/\partial t)\, \delta s \equiv \equiv c^{-2}\, I \int_L (\partial V_1/\partial t)\, \delta l. \tag{5}$$

Substituting in eqs. (4) and (5)

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$$dA_1 = (j dz/4\pi r)\, u, \; dV_1 = c^2 q dz/4\pi r, \qquad (6)$$

with u denoting the unit vector of the z axis and r the distance from the dz element **, yields as was announced,

$$d(F_{1z} + F_{2z}) = 0 \, . \qquad (7)$$

Finally the dilemma is:

I. The equality of action and reaction and the equivalence of the Coulomb [4] and Ampère pictures of magnetism hold, in which case the new, non-conventional, force [5] must exist.

II. The force [5] does not exist, in which case there is neither equality of action and reaction nor equivalence of the Coulomb [4] and Ampère pictures of magnetism.

Incidentally, the problem of existence or non-existence of the force $c^{-2}\, di/dt \oint V_1\, \delta l$ can be cast in the form of a Gedankenexperiment parallelling this one.

References

1. L. Golub, J. P. Carrico, T. S. Stein and M. C. Weisskopf, Phys. Letters 25A (1967) 495. One thing that makes the proof questionable is that in our paper [2] the magnetic moment change follows the intensity change in a fixed circuit, while this is obviously not the case in Golub s experiment.
2. O. Costa de Beauregard, Phys. Letters 25A (1967) 95.
3. O. Costa de Beauregard, Phys. Letters 24A (1967) 177.
4. O. Costa de Beauregard, Compt. Rend. 264 B (1967), 731.

** For given r and $\Delta t \equiv r/c$, dA_1 and dV_1 and dV_1 satisfy the Lorentz condition.

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