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INTERSUBJECTIVITY, RELATIVISTIC INVARIANCE, AND CONDITIONALS (CLASSICAL AND QUANTAL)

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Mutual dependence of probabilities of two physical occurences implies existence of an interaction or causal link between them. Axiomatized along these lines, the formalization of conditionals comes out as identical to that of transition probabilities.

Key words: joint, conditional, and transition probabilities.

1. LOGIC, SPACETIME INVARIANCE, AND PROBABILITY

The contention here is that probability is not the affair of logic and algebra alone. Insofar as *chance occurrences* are *events* observable in space and time, *geometry* is implied also, and so the *invariance* required in the description is exigible of the probability scheme itself.

The *joint probability* of non-independent chance occurrences expresses their *physical interaction.* In saying this are we (as Jaynes [1] would put it) "unduly confusing epistemology and ontology"? Nay: Logical inference is a thought causality following, so to speak, a thought telegraphic wire. For example, if at Berlin a ball is placed in one of two boxes sent the one to Atlanta and the other to Cape Town, the inference drawn by each recipient as to what the other one finds follows the spacetime ABC or CBA zigzag; therefore geometric invariance is required in logical inference no less than it is in physical causality.

So, as emphasized by Accardi [2], the concepts of a *joint* or of a *conditional* probability on the one hand, of a *transition* probability on the other hand, are tied to each other and should be amenable to a common formalization.

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As *causality* and existence of an *interaction* are synonymous, the *propagation of causality* should be displayed in the very formalization of probabilities—and it will be so in a geometrically invariant one.

Thus emerges the concept of a *geometric theory of physical probabilities* which, very happily, comes out as identical to the existing one of *transition probabilities.*

Finally, as there is agreement between observers of an event, *intersubjectivity* rather than (questionable) objectivity or than (inappropriate) subjectivity qualifies the *physical* probabilities.

2. THE GEOMETRIC THEORY OF PHYSICAL PROBABILITIES

The well-known Bayes-Laplace expression of the (essentially symmetric) *joint probability* of two non-independent occurrences A and C, in terms of their *inverse conditional probabilities* and of their *prior probabilities* is (dropping for brevity the familiar P symbols)

$$|A) \cap (C| = |C) \cap (A| = |A|C)(C| = |A)(A|C| = |C|A)(A|. \qquad (1)$$

Unexpressed in this formula is a most important information: *geometric invariance* and *action-reaction reciprocity* of causality.

Therefore, rather than two inverse or "relative" conditional probabilities, let us use *one intrinsic reciprocal conditional probability* "of A if C or of C if A:

$$(A|C) = (C|A), \qquad (2)$$

and have *both* priors displayed together in the formula; and, indeed, is not the very epithet "joint" an invitation to do so?

Thus, we replace the expression (1) of the joint probability by that of the (un-normalized) *joint number of chances*

$$|A) \cdot (C| = |C) \cdot (A| = |A)(A|C)(C| = |C)(C|A)(A|. \qquad (3)$$

Identification of the concepts expressed in (1) and (3) is forbidden by normalization constraints. In both cases, (1) and (3), the prior probabilities $|A)$ and $(C|$ we normalize to unity. Then, if in (1) the inverse conditionals are normalized via

$$S_A\,|A|C) = 1, \quad S_C\,|C|A) = 1, \qquad (4)$$

the joint probability comes out as normalized via

$$SS\,|A)(C| = 1. \qquad (5)$$

In (3), instead, we normalize our "reciprocal conditional" via

$$S_A (A|C) = S_C (C|A) = 1; \qquad (6)$$

identification of (1) and (3) is unacceptable as entailing $(A|C) = \delta_{AC}$, that is, independence of A and C.

This formalism, which contains the group-generating formula

$$(A|C) = S (A|B)(B|C) \qquad (7)$$

(where the summation is over mutually exclusive states) is none other than the one of *physical transition probabilities.*

In a spacetime picture, (7) is the propagators composition formula; $(A|B)$ is the "mutual cross section", $|A)$ and $(B|$ are the "occupation probabilities" (initial one of an initial state, final one of a final state), and $|A) \cdot (B|$ the "dressed collision probability."

Expressed either in a spacetime or a momentum-energy picture, this formalism has "manifest Lorentz invariance." It also has PT invariance à la Loschmidt, *here derived directly from probability reversal.* It also has prediction-retrodiction symmetry; and, for chained events $ABCD\ldots$, it has topological, or "zigzagging causality invariance."

Concatenations with more than two propagators attached to a vertex can be used as guidelines for computing collision probabilities.

Let us illustrate by two examples the "manifestly covariant" and "intersubjective" handling of correlated chance occurrences, and how "conditionality" is built in the transition probability concept.

Two Spacelike Separated Occurrences: In 1987, 15 neutrinos of the swarm emitted by a galactic supernoval explosion were detected in laboratories, some of which are thousands of miles apart. What sense does it make to express the joint probability of two such detections by formula (1)? Quite the contrary, use of formulas (3) and (7) expresses all right, by $(B|A)$ and $(B|C)$, the propagation of neutrinos from the source B to the receptors A and C, and by $|A)$ and $(C|$ the detection efficiencies. Conditionality consists in that the formula holds *if* appropriately adjusted detectors are present at A and C—and it is reciprocal.

Two Timelike Separated Occurrences: What is the joint probability that, having seen a lightning, I will hear the corresponding thunder roar? Bayes' formula (1) gives for it two "inverse" answers, a predictive and a retrodictive one—not severing, however, propagation from detection of the signals. Lightning is not the cause of thunder: both proceed from a common earlier cause, an electric discharge emitting "jointly" photons and phonons.

Having sighted a flash of "summer lightning," I will not hear the thunder, if only because, when the phonons arrive, I will have lost interest in them. And if indoors, at night, I hear the thunder, I will not have seen the lightning. My eye and ear acutenesses are both expressed by the priors; and reciprocal conditionality is expressed as $(A|C) = (C|A)$. Using a pre-field-theoretic idea of "causality," Bayes' formula ignores these points.

In view of the following, strong emphasis is needed on one point of this logic of classical probabilities: The intermediate summations $|B)(B|$ in formula (7) are thought of as ranging over (possible) "real hidden states." *This is a trait inherent in the realistic natural philosophy.*

That a built-in "correspondence" exists between this renovated probability scheme, and the radically novel "wavelike" one invented in quantum mechanics by Born and by Jordan, I deem highly significant.

3. WAVELIKE PROBABILITIES, CPT INVARIANCE, QUANTAL NONSEPARABILITY

Einstein and de Broglie's wave-particle dualism, a Copernican breakthrough, together with its "statistical interpretation" by Born [3] and by Jordan [4], is a wonderful problem-solving toolkit. But as a paradigmatic revolution it causes headaches to the natural philospher.

The Born-Jordan wavelike calculation recipe is: Add partial, and multiply independent, *amplitudes* rather than probabilities. Interpretors quarrel here, as Bitsakis [5] summarizes. One school says this is no revolution at all, that only an appropriate handling of conditionals is needed. The other school, to which I adhere, holds that Born and Jordan's is a *non-Laplacian probability scheme.*

Born's seemingly innocent proposal that the wave's *intensity* expresses the particle's position *probability density* undermines the very concept of a reality. This it does via *wavelike interference*, associated with (strong) *spacetime reversibility.*

Corresponding to the "reversible transition probability" (2) there is the *self-adjoint transition amplitude*

$$< A|C >=< C|A > *. \tag{8}$$

Strong spacetime reversal is cryptically formalized in this Hermitian symmetry. In the spacetime picture, $< A|C >$ is a propagator, the PT reversal of which obviously is

$$PT: \ < A|C > \rightleftharpoons < C|A > . \tag{9}$$

Particle-antiparticle exchange can be expressed as

$$C: \; < A|C > \rightleftharpoons < A|C > *. \qquad (10)$$

Thus, the geometric rendering of the *Hermitian symmetry* (8) is the *strong spacetime reversal*: $CPT = 1$.

Corresponding to the "joint number of chances" (3), there is the joint amplitude

$$|A > \cdot < C| = |A >< A|C >< C|, \qquad (11)$$

and corresponding to the generating formula (7) of Markov chains, there is that of Landé [6] chains

$$< A|C > = S < A|B >< B|C > . \qquad (12)$$

Concatenations with more than two propagators at one vertex are known as "Feynman graphs."

Algebraic *non-separability* and geometric *non-locality*—a paradigmatic revolution—result from the expression of the (reversible) transition probability

$$(A|C) = < A|C >< C|A > = | < A|C > |^2; \qquad (13)$$

insertion of (12) in it generates cross or "interference" terms. This *interference of probability amplitudes* precludes that the "intermediate sums" over $|B >< B|$'s be conceived as over "real hidden states"—as were the classical $|B)(B|$'s.

For this very reason, there is no quantum analog to the one ball and two boxes problem: the logical inference drawn from Atlanta to Cape Town or vice-versa does indeed follow the ABC or CBA zigzag, but at Berlin not a ball, only an Alice-in-Wonderland smile-of-a-ball was hidden!

Wheeler's [7] metaphor for this is the "smoky dragon." For example, a photon prepared as polarized along A and measured as polarized along C has inbetween a "smoky" amplitude $\cos B = \cos A \cos C + \sin A \sin C$, with $B = C - A$. This is easily tested by inserting, and arbitrarily rotating, a birefringent crystal. Clearly, the measured "polarization state" $|C >$ *cannot* have preexisted to its measurement. Thus, "delayed choice" of the angle C does display "retrocausation."

Similarly, in an EPR [8] correlation, linear polarizations measured the one as $|A >$, the other as $|C >$, did not preexist as such in the source B; there was, coiled in B, a (twin mouthed) smoky dragon.

A time-reversed EPR correlation is analogous to a two-slit interference; as one cannot retrodict from which slit came any detected photon, coiled in the sink B waits a (twin tailed) dragon.

These are three topologically equivalent cases, with respective spacetime $<$ (or C), V, or $\wedge$ shapes of the ABC zigzag.

I feel quite sure [9] that the forthcoming "timed EPR experiments" will evidence the zigzagging, reversible, CPT invariant microcausality: Varying independently the AB and BC distances will display the spacetime geography of the ABC zigzag.

Retrocausation, as evidenced in the delayed-choice experiments, does of course not imply that "one can kill his grandfather in his cradle"! Lawlike reversible microcausality is one thing; factlike irreversible macrocausality is something else. Non-separability, nevertheless, is fatal to the concept of a macro-reality.

There have been significant advances in the so-called measurement problem, emphasizing the role of "destructive interference." Zurek [10], and others [11], have shown how deliberate ignorance of those apparatus variables that are uncoupled to the measured magnitudes blurs the off-diagonal terms.

Of course it is trivial [12,13] that assignment of a probability depends upon what one chooses to know or not, to control or not. And, or course, statistical averaging generates no more than the symbol of a *trompe-l'oeil* sort of reality. Here the implication is still more radical.

So, after all, even the grasped tail and the biting mouth of Wheeler's dragon may not "really" hang "down here"! It may be that physical intersubjectivity, being equally distant from objectivity and subjectivity, has, in the generation of chance occurrence, a more active role than is presently recognized.

4. RECIPROCAL CONDITIONALITY VERSUS HIDDEN REALISM

Let us go back to Einstein's [8] reality criterion: "If, without in any way disturbing a system, one can [exactly] predict [teledict would be more apposite] the value of a physical quantity [by measuring a strictly correlated one], then there must exist a [corresponding] element of reality." The technical tool EPR used was the non-relativistic Schrödinger formalism; the example chosen was position x or momentum p measurement for particles a and b correlated via the commuting operators $x_a - x_b$ and $p_a + p_b$. Instead, we choose here linear polarizations of paired photons, because then the formalism can easily be made Lorentz and CPT invariant.

Experimentation disproves that finding at A the result $|A>$ reveals existence at C of the strictly correlated state $|C>$: Linear polarizations *can* be *independently* measured at A and C, and the amplitude $<A|C>$ of two YES answers is A and C *symmetric*. So, which of the two measurements collapses the other state?

Einstein's realistic assumption thus contradicts the very spirit of quantum physics. What is at stake is a doubly conditional probability

$(A|C)$: *If* the question $|A>$ is put at A, *and if* the question $|C>$ is put at C, *then* the amplitude $<A|C>$ is expressed by formula (7). *Reciprocal conditionality, not "hidden realism", is operational.*

5. REVERSIBLE CAUSALITY AS IDENTIFIED WITH RECIPROCAL CONDITIONALITY

The hint implicit in Laplace's 1774 "Memoir on the Probability of Causes" finds an answer in relativistic quantum mechanics. To such an identification Jaynes [1] objects that it "unduly confuses ontology and epistemology," adding that "in pure deductive logic, if A implies B, not $-B$ implies not $-A$; [but] if we tried to [identify implication with physical causation] "we could hardly accept that not $-B$ is the physical cause of not $-A$." What Jaynes objects to is retrocausation; defining not $-A$ as *observation of not A* (not the weaker "A non-observed") it is clear that, in a dichotomic context, if A causes B, not $-A$ causes not $-B$. So, to falsify Jaynes' statement, it suffices to produce an example where there is both dichotomy and reversibility. Here is one: A low intensity laser beam crosses in succession two birefrigent crystals a and b with parallel axes; by A and not $-A, B$ and not B, we denote the two possible answers.

If the two crystals are independently oriented, although the context is no more deterministic, reversibility still holds. If the beam is very long, there is plenty of time for fixing the orientation at b after a photon has passed a; that is, a delayed choice is possible among *incompatible* questions put at b. This is operational proof that, in the non-Laplacian scheme, retro-inference expresses retrocausation.

In the quoted paper, Jaynes comes very near to accepting (after work done by Gull) that in today's state of the art the only acceptable hidden-variable theories are those where causality is time-reversible, so that "exorcism of the superluminal spook" appears as canonization of the "teleological spook." Let me recall that an (informal) group of theoretists is playing, since quite a few years, with a yet unrefuted model combining these two features [14].

6. TO CONCLUDE

The contention here is that logic and spacetime kinematics are not separable from each other, so that *probabilistic inference* is a symbolic rendering of *causality*.

We have argued that the idea of *conditionality* is implied in the *transition probability* concept of physics, which we propose as an improvement over the Bayesian joint probability concept.

A Landé-style correspondence [3] exists between this scheme and the Born-Jordan wavelike probability scheme of quantum mechanics. Beyond obvious incidences concerning the idea of reality, this may have also significance concerning the handling of probability in general: "The greatest simplicity and elegance is attained when only square integrable functions are admitted." [15]

Chance, as it seems to me, is a *physical* concept. If the basic chance game is the quantum one, then (as Landé also argues) the most advanced form of the "calculus of probabilities" is Born and Jordan's "wavelike" one.

These ideas I have presented a few times [16,17], but two key elements, emphasized in this paper, I did not then make sufficiently clear.

The one has to do with Jayne's [1] radical distinction between "epistemology" and "ontology," which I do question insofar as chance *occurrences* are observable, physical *events*.

The other point having raised questions [18] is that substituing the *transition probability* to the Bayesian *joint probability* modifies not only the formalization, but also the conceptualization.

Comments made in Paris, at the MAXENT 1992 Conference after my presentation, by Skilling, Garrett, Gull, and also by Frohner have been extremely helpful to me regarding these two points.

The philosophy underlying this work is that physics is essentially probabilistic, so that there is equivalence and reciprocity of the *information* and the *negentropy* concepts. Then Wigner's [19] assumption of a two-way mind-matter interaction makes sense. But this I will not delve into here.

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