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Induced Electromagnetic Inertia and Physicality of the 4-Vector Potential

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Abstract

It is claimed (our basic statement) that a point charge Q *evolving inside a 4-potential* $A^i(x^j)$ *expressed in the source-adhering gauge receives an induced 4-momentum* QA^i *evidencing the reaction of* A*'s sources to any force applied to* Q. *Relativistic far-action–reaction between four-dimensional line elements à la Wheeler–Feynman is upheld. The 6-field* B^{ij} *present in forces and the 4-potential* A^i *present in energies and momenta are held equally physical, because the integration condition is fixed via mass–energy equivalence and action–reaction opposition. This is taken as an* ex absurdo *proof that the photon has a rest mass. As an example, the interaction between a heavy toroidal magnet at rest in the laboratory and a flying point charge is discussed. Connections with the Aharonov–Bohm effect and de Broglie's electron diffraction are examined. Crucial tests of the basic statement using a toroidal magnet are proposed.*

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Key words: source-adhering gauge, 4-potential, Einstein's mass–energy equivalence, Curie–Stern–Gerlach force, Aharonov–Bohm effect, Meissner effect, photon rest mass

1. INTRODUCTION: AN UNCONVENTIONAL BUT TESTABLE CLAIM

The basic statement (BS) we intend to prove is this: Einstein's energy–mass equivalence and (with a relativistic meaning to be stated) action–reaction opposition between sources of the field uniquely select the electromagnetic gauge as an integration condition.[1,2] Consequently, a point charge of value Q evolving inside a 4-potential A^i ($i,j,k,l = 1,2,3,4$; $x^4 = ict$) expressed in the source-adhering gauge (say, Coulomb's in electrostatics) has an *inertially induced 4-momentum* QA^i.

The logic supporting these claims is as follows: while electromagnetic forces (linear or angular) are *field-dependent*, their integrals over space or time, energies, and momenta are *potential-dependent*. Since the integration conditions are physically evidenced via the conservation laws, the 4-potential is (on par with the 6-field) a measurable magnitude.

There is much similarity between selection of the electrostatic gauge via the conserved canonical energy $c^2m + QV$ and selection of the magnetostatic gauge via the conserved canonical momentum $m\mathbf{v} + Q\mathbf{A}$. Such control of the 4-potential by mechanical inertia hints at the existence of a *sui generis* electrogravity coupling which is transparent in some of our formulas. This line of inquiry is not pursued here. Rather, convergence of our conclusion with an independent claim,[3] stating that consistency of quantum electrodynamics demands that the Lorentz condition be a field equation, is stressed.

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The overall conclusion then is that the photon has a rest mass, a claim also upheld by Evans and Vigier[4] from very different premises.

2. SELECTION OF THE GAUGE VIA MASS–ENERGY EQUIVALENCE

Clear as midday and for that reason overlooked is selection of the Coulomb gauge via Einstein's energy–mass equivalence. By assuming the classical electron mass to be of electric origin and expressing it as $-1/2$ eV, we can uniquely select the Coulomb gauge. Also, the weight of a vessel containing gaseous or liquid hydrogen includes the atomic mass defect. So no arbitrariness is allowed in the electric potential. It is surprising that so well substantiated a fact, with such important consequences, is so radically overlooked.

As emphasized by de Broglie[5] using a specific example and by Brillouin[6] in a book, the implication is that the effective mass M of a point charge of value Q and mechanical mass m immersed in an electric potential $V(\mathbf{r},t)$ *expressed in the Coulomb gauge* is

$$M = m + c^{-2}QV. \qquad (1)$$

The idea had originated earlier, in the pre-Maxwellian electrodynamics of Weber[7] and of Clausius,[8] where c was defined as the ratio of the emu to esu charge unit (perhaps up to some multiplicative factor).

So suspended from a dynamometer in a gravity field g, or running on the plate of a balance, a charged particle's weight is measured as $M\mathbf{g}$. Therefore, according to Einstein's inertia–gravity equivalence, accelerated *by any means* the particle responds inertially via the effective mass M. Brillouin's *effective momentum* is $\mathbf{P} = m\mathbf{v}$.

Thus everything goes on[9] as if the accelerated particle were feeling an *inertially induced electric field*

$$\mathcal{E} = c^{-2}V\mathbf{g} \quad \text{or} \quad \mathcal{E} = c^{-2}V\mathbf{r}''. \tag{2}$$

3. SELECTION OF THE GAUGE VIA ACTION–REACTION OPPOSITION

Postponing comments on the relativistic action–reaction formalism of Wheeler and Feynman,[10,11] we now propose an elementary proof of our basic statement in the instantaneous, low-velocities approximation.

Starting from rest at time zero in an inertial frame two r distant point charges Q_a and Q_b pick up collinear accelerations $\mathbf{a}''$ and $\mathbf{b}''$, but their effective masses M_a and M_b differ from the mechanical masses m_a and m_b because in addition to, say, b's Coulomb field, charge a also feels b's Faraday field, $-\partial\mathbf{A}/\partial t$, induced by the acceleration $\mathbf{b}''$.

Since the Coulomb potentials generated at each charge by the other one are such that

$$Q_aV_a = Q_bV_b = Q_aQ_b|\mathbf{r}_{ab}|^{-1}, \tag{3}$$

and since the associated magnetic potentials are

$$\mathbf{A}_a = V_a\mathbf{b}', \quad \mathbf{A}_b = V_b\,\mathbf{a}', \tag{4}$$

the coupled equations of motion are

$$\mathbf{P}_a + \mathbf{P}_b \equiv (m_a\mathbf{a}' + Q_a\mathbf{A}_a) + (m_b\mathbf{b}' + Q_b\mathbf{A}_b) = \mathbf{0}, \tag{5}$$

or equivalently, thanks to (1) and (4),

$$\mathbf{L}_a + \mathbf{L}_b \equiv M_a\mathbf{a}' + M_b\mathbf{b}' = \mathbf{0}. \tag{6}$$

The conserved total linear momentum is thus expressed as *zero* (in the inertial frame where the particles started from rest) provided that the Coulomb gauge is selected. Linear action–reaction opposition holds iff each particle is endowed with the potential momentum $Q\mathbf{A}$ and the potential mass $c^{-2}QV$, our basic statement.

4. COVARIANT ELECTROMAGNETIC FORMULAS

Generalizing $\mathcal{E}$, the inertially induced electromagnetic 6-field is (the derivatives referring to proper time)

$$\mathcal{B}^{ij} := -c^{-2}(A^ix''^j - A^jx''^i). \tag{7}$$

A point charge thus feels the inertially induced 6-force $F^{ij} = Q\mathcal{B}^{ij}$ and the corollary 4-force

$$F^i := F^{ij}x'_j = -c^{-2}QA^kx'_kx''^i. \tag{8}$$

At first order in v/c the 3-space projection of F^i is (the derivative referring to Newtonian time)

$$\mathbf{F} = (c^{-2}QV - \mathbf{A}\cdot\mathbf{v})\mathbf{r}''; \tag{9}$$

the first term in the parentheses is the previous $Q\mathcal{E}$; operationality of the second term is displayed in formula (30) below.

Evidencing the BS, two different Minkowskian formulas extend formula (1):

(1) The effective 4-momentum of a point charge Q of rest mass m_0 and 4-velocity x'^i ($x'_ix'^i = -c^2, x'_ix''^i = 0$) immersed in a 4-potential $A^i(x^j)$ expressed in the source adhering gauge is

$$P^i := m_0x'^i + QA^i. \tag{10}$$

(2) The particle's effective proper mass,

$$M_0 := m_0 - c^{-2}QA_kx'^k, \tag{11}$$

enters the effective longitudinal 4-momentum (such that $P_iL^i = L_iL^i$),

$$L^i = M_0x'^i; \tag{12}$$

the potential contribution in (11) generates the inertial force (6).

An *electric* or a *magnetic* phenomenology is expressed by these formulas according to whether A^i is timelike and V dominates, or A^i is spacelike and $\mathbf{A}$ dominates.

In view of the following we recall the Hamilton–Jacobi variational formula yielding[12] the Lorentz equation of motion of a point charge immersed in a given 4-potential A^i:

$$0 = \delta\mathcal{A} := \delta\int P_i\,dx^i = \delta\int L_i\,dx^i. \tag{13}$$

While the (timelike) p^i trajectories are uniquely specified, the P^i lines are not, since P^i is defined up to a 4-gradient. The p^i lines, oblique on the Hamilton–Jacobi 3-surfaces, are "anomalous rays," in the wording of optical anisotropic media. De Broglie's formula $\mathcal{A} = \hbar\varphi$ likens *canonical action* to *phase*, whence the particle's velocity to the wave's group velocity. The phase velocity is x^i and gauge-dependent.

5. RELATIVISTIC FAR INTERACTION IN THE WHEELER–FEYNMAN MODEL

Action–reaction opposition is often deemed nonsense in relativity theory because (1) Newton's "universal" simultaneity is rejected, and (2) interaction is via fields carrying energy, momentum, and angular momentum.

However, a perfect Minkowskian formalization and conceptualization of action–reaction is allowed under three assumptions: (1) all emitted energy, momentum, and angular momentum are subsequently absorbed, implying a far-action rendering of the conservation laws; (2) not instantaneous of course, the interaction is between line elements of space-time trajectories; (3) the propagation law is PT-reversible (CPT-reversible in quantum theory).

Wheeler and Feynman's two famous papers[10,11] propose such a model of electromagnetism, a four-dimensional transcription of the magnetostatics of currents, in fact.[13]

The amperian $r^{-3}\mathbf{r}$ dependence goes into a $\delta(r^2)r^i$ dependence; the magnetic field goes into the electromagnetic field; the Grassman 3-force (linear density) goes into the Lorentz 6-force; the "shortened Ampère force" (linear density) goes into an amperian 4-force; the amperian stress tension[14] along a wire goes into the particle's 4-momentum.

The space-time trajectory of a point charge is thus viewed as an open timelike current. The whole scheme transcribes the statics-of-filaments rendering of magnetostatics. Mutual *energy* goes into inter*action*.

Wheeler and Fenyman's 4-potential created at particle a by particle b is $r^i \equiv a^i - b^i$ denoting their mutual space-time distance and the integral being over b's trajectory,

$$A_a^i = Q_b \int \delta(r^2) db^i; \tag{14}$$

this transcribes the amperian $\mathbf{A} = i \oint r^{-1} \mathbf{dl}$.

The coupled equations of motion of particles a and b evidencing direct far-action-reaction (no mention of the Lorentz force) are

$$d^2P_a^i = -d^2P_b^i = Q_a Q_b\, \delta'(r^2) da_k db^k r^i, \tag{15}$$

with the P^i's defined in (10). Thanks to $\delta'(r^2)$, the *formal* far-interaction between *any* two line elements acts *only* for lightlike separations (so there is no self-interaction of a particle). Wheeler and Feynman's action-reaction formula transcribes the shortened Ampère force formula $(\mathbf{dT}/dl)_a = -(\mathbf{dT}/dl)_b = r^{-3}(\mathbf{dl}_a \cdot \mathbf{dl}_b)\mathbf{r}$ between two current elements, with $\mathbf{T}$ denoting[14] the stress tension along the wire.

Selection of the source-adhering gauge (14) is via action-reaction opposition (15), plus something important, our BS: each particle is endowed with an extra inertia QA^i, meaning that it feels the inertial "Faraday force" QA''^i.

An alternative expression of the 4-momentum conservation equation,

$$d^2(P_a^i + P_b^i) = 0, \tag{16}$$

is

$$d^2(L_a^i + L_b^i) = 0, \tag{17}$$

obtained by substituting in the expressions (10) of P_a^i and P_b^i the A^i's given by (14) and factoring da^i and db^i.

6. FLATTENING THE LIGHT CONE AND RECOVERING INSTANTANEITY

If both particles move slowly (relative to each other and in the reference frame), in the limit $c \Rightarrow \infty$ formula (14) goes into

$$\mathbf{A}_a = r_{ab}^{-1} Q_b \mathbf{b}', \quad V_a = r_{ab}^{-1} Q_b, \tag{18}$$

the derivative $\mathbf{b}'$ referring to Newtonian time.

Formula (15) expressing action-reaction opposition goes into

$$\mathbf{P}_a' = -\mathbf{P}_b' = Q_a Q_b r_{ab}^{-3}(1 - c^{-2} a' b')\mathbf{r}_{ab}, \tag{19}$$

where the $\mathbf{P}$'s are defined in (5). In the preferred zero momentum inertial frame

$$\mathbf{P}_a + \mathbf{P}_b = 0, \tag{20}$$

or equivalently,

$$\mathbf{L}_a + \mathbf{L}_b = 0, \tag{21}$$

the $\mathbf{L}$'s being defined in (6).

Formulas (18) to (21) belong to the Clausius[8] electrodynamics, which is derivable[15] from them.

So, there is direct far-action-reaction under the BS condition: each particle is endowed with an induced momentum $Q\mathbf{A}$ expressed in the source-adhering gauge. Of course, accelerated by any means inside a 6-field a point charge accelerates the sources and feels a reaction. The point is that this reaction is *exactly* $Q\mathbf{A}$.

7. HEAVY TOROIDAL MAGNET AT REST AND FLYING POINT CHARGE

Consider, fixed in the laboratory's $(\mathbf{r},t)$ frame, a toroidal magnet $\mathcal{M}$ of trapped flux Φ and, flying inside its curlless potential $\mathbf{A}(\mathbf{r})$, a test point charge $\mathcal{C}$, say an electron of charge $-e$ and mass $m = m_0(1 - \beta^2)^{-1/2}$.

Let us prove that energy and momentum conservation each entail our BS: the electron does not move inertially but according to the equation

$$\mathbf{P} \equiv m\mathbf{v} - e\mathbf{A} = \text{const}, \tag{22}$$

where the gauge is the source-adhering gauge [Eq. (25) below]. It is thus accelerated by the *Faraday force* (straight d's)

$$\mathcal{F} = Q\mathcal{E}, \quad \mathcal{E} = -\mathrm{d}\mathbf{A}/dt. \tag{23}$$

This is because the *moving* charge's magnetic field $\mathbf{B}$ injects *inside* the magnet a Maxwell energy density, and its electric field $\mathbf{E}$ a Poynting momentum density, both transferred to the laboratory equipment. Since neither energy nor momentum appear in the vacuum where $\mathcal{M}$'s field is zero, the electron must recoil.

Wesley,[16] a "dissident physicist," derives this same conclusion from Weber's electrodynamics and terms the phenomenon "motional induction."

Conservation of the flying electron's energy, the fourth equation of motion, will come out as

$$c^2m - e\mathbf{A}\cdot\mathbf{v} = \text{const} \quad \text{or} \quad P_4 - L_4 = \text{const}. \tag{24}$$

Idealizing the magnet $\mathcal{M}$ as a (not necessarily circular) filament carrying per line element a magnetic moment $d\mathbf{N} = \Phi\, d\mathbf{l}$, Φ constant, we express its source-adhering 4-potential as

$$\mathbf{A} = \Phi \oint r^{-3}\mathbf{r} \times d\mathbf{l}, \quad V = 0. \tag{25}$$

Then $-\mathbf{r}$, denoting the vectorial half retarded, half advanced distance from the flying electron $\mathcal{C}$ to any point on the magnet $\mathcal{M}$, the electromagnetic field created on $\mathcal{M}$ by $\mathcal{C}$ is, at first order in v/c,

$$\mathbf{E} = -er^{-3}\mathbf{r}, \quad \mathbf{B} = c^{-2}\mathbf{E} \times \mathbf{v}. \tag{26}$$

The *inhomogeneous* field $\mathbf{B}$ exerts upon each line element $d\mathbf{l}$ of the magnet a force $\Phi\partial(\mathbf{B}\cdot d\mathbf{l})$ (a very real force, computable from the Lorentz force if $\mathcal{M}$ is a solenoid). It is the gradient of the energy $\Phi\mathbf{B}\cdot d\mathbf{l}$ injected *inside* $\mathcal{M}$ by the *moving* charge (the "maximizing flux" force if M is a solenoid).

Expanding the expression $(\partial \times \mathbf{B}) \times d\mathbf{l}$, noticing that $\oint(d\mathbf{l}\cdot\partial)\mathbf{B} \equiv \oint d\mathbf{B} = 0$, and using Maxwell's equation $\partial \times \mathbf{B} = \partial\mathbf{E}/\partial t$, we get the Poynting-style expression (in mixed units) of the (kinematically unobservable) momentum picked up by the magnet:

$$\mathbf{p}_m = \Phi \oint \mathbf{E} \times d\mathbf{l}, \tag{27}$$

the 1967 "hidden momentum in magnets."[17-19]

As the electron picks up the momentum

$$\mathbf{p}_c = e\mathbf{A}, \tag{28}$$

the system's *conserved total momentum* is equated to zero iff the source-adhering gauge (24) is selected:

$$\mathbf{p}_c + \mathbf{p}_m = \mathbf{0}. \tag{29}$$

Since the magnet's external electromagnetic field is zero, the vacuum contains neither Maxwell energy nor Poynting momentum. Residing entirely within the bodies, the system's mutual energy is

$$\begin{aligned} W(t) &\equiv -e\Phi \oint \mathbf{B}\cdot d\mathbf{l} \\ &= -e\Phi \oint r^{-3}\,(\mathbf{v}\times\mathbf{r})\cdot d\mathbf{l} = -e\mathbf{A}\cdot\mathbf{v}. \end{aligned} \tag{30}$$

Two equivalent expressions of the mutual energy are thus displayed: an exact, gauge-independent expression, localizing it in the magnet, and a gauge-dependent expression valid at first order in v/c, localizing it in the electron generating the familiar energy $W = I\oint \mathbf{A}\cdot d\mathbf{l}$ in an amperian current.

Since the heavy magnet does not recoil, the electron's equations of motion are (22) and (24). Q.E.D.

Returning to the action integral (13), we notice that as a consequence of (22), the 3-space contribution is zero. But according to (11), the time contribution, taking care of the velocity dependence of the mass m, is at first order in v/c

$$\mathcal{A} = \oint W dt = -e\oint \mathbf{A}\cdot\mathbf{v}\, dt = -e\oint \mathbf{A}\cdot d\mathbf{r}, \tag{31}$$

the formula used in the Aharonov–Bohm effect. The "anomalous ray" aspect of the trajectory shows up in the fourth dimension.

8. ELECTRON DIFFRACTION IN THE PRESENCE OF A TOROIDAL MAGNET

De Broglie's[20] formula relating the particle's canonical 4-momentum to the wave's 4-frequency

$$P^i \equiv p^i - eA^i = \hbar k^i \tag{32}$$

is vindicated in the Aharonov–Bohm effect, tested as gauge-invariant by use of a closed integration contour. But this may well be an artifact. The arbitrary gradient is *unobservable*, and its insertion in (32), where $\mathbf{p}$ and $\mathbf{k}$ are both measurable, is a nuisance. Should we believe that the ticking of de Broglie's clock is not intrinsic?

Let us prove that in the identical absence of an electromagnetic field, but in the presence of a toroidal magnet, crystal electron diffraction selects the source-adhering gauge (25) via

$$B^{ij} \equiv 0 \Rightarrow A^i_\infty \equiv 0 \tag{33}$$

by yielding the result

$$\mathbf{P} \equiv \mathbf{p}(\mathbf{r}) - e\mathbf{A}(\mathbf{r}) = \hbar\mathbf{k} = \text{const.} \tag{34}$$

Together with (22), (24), and (25) is an equation of the problem. As the magnet levies the energy and momentum tolls previously discussed, electrons shot by a "gun" at voltage V have the anomalous kinetic energy (mixed units)

$$c^2(m - m_0) = -e(V + \mathbf{A}\cdot\mathbf{v}). \tag{35}$$

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Correlatively, ~~the BS~~ states that there is the 3-frequency shift

$$\hbar\Delta\mathbf{k} = -e\mathbf{A}. \tag{36}$$

This is an observable effect depending on the relative positions of gun and magnet, but not of crystal and magnet.

With a toroidal magnet coaxial to the gun and crystal a clearcut quantitative test is possible; the "anomalous ray effect" occurs in the fourth dimension. With a magnet *not* coaxial to the gun, the "anomalous ray" would show up in 3-space.

9. MEISSNER EFFECT EVIDENCING THE BASIC STATEMENT

Circular electron trajectories coaxial with an infinitely long straight magnet are allowed, obeying the equation

$$\mathbf{P} = 0 \quad \text{or} \quad m\mathbf{v} = e\mathbf{A}, \tag{37}$$

which explicitly displays the BS. The "anomalous rays" are helixes around the time axis.

This is akin to "vacuum superconduction." The *canonical action* $\mathcal{A}$ around the circle is zero, and the *potential action* is quantized according to

$$\mathcal{A} = -2\pi rA = nh. \tag{38}$$

Superconduction proper in a circular ring coaxial with the magnet is the guided wave version of the effect and conclusive proof of the BS. Since the magnet's flux is trapped, the ring feels no force, so Eq. (37) (with $2e$ replacing e) is obeyed, together with

$$W = I\Phi = nh\nu, \quad \Phi = -nh/2e, \quad \mathrm{I} = -2e\nu. \tag{39}$$

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Z / 10. MODIFIED LENS EFFECT RECIPROCAL TO FARADAY INDUCTION

Reversing the flux Φ in a toroidal magnet interlaced with a circuit of resistance R induces the well-known translation of an electric charge Q. The equation $RQ = 2\Phi$ holds as a consequence of $I\Phi' = RI^2$ and $I = Q'$.

Conversely, the BS states that the time delay from application of a voltage V at the circuit's terminals to inception of a steady current depends on the direction in which the current is sent. The two curves displaying the rise of the intensity $I(t)$ should build up a contour resembling a hysteresis loop, with the enclosed area measuring the translated charge. The difference between the two energies needed is $2I\Phi = RIQ$, whence $RQ = 2\Phi$.

11. CONCLUSION

Proofs have been adduced of the following basic statement: a point charge Q evolving inside a 4-potential A^i expressed in the source-adhering gauge is endowed with an induced extra momentum QA^i. Thus on par with the 6-field, the electromagnetic 4-potential is a measurable magnitude.

Crucial tests involving a toroidal magnet have been proposed.

The whole argumentation, upheld as an *ex absurdo* claim that the photon has a rest mass, corroborates an independent claim[3] stating that consistency of quantum electrodynamics demands that the Lorentz condition be a field equation.

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Résumé

On soutient (énoncé fondamental) qu'une charge ponctuelle Q *évoluant dans un 4-potentiel* $A^i(x^j)$ *exprimé dans la jauge adhérente à la source reçoit une* 4-impulsion induite QA^i *mettant en évidence la réaction des sources de* A *à toute force appliquée à* Q. *L'action-réaction relativiste à distance à la Wheeler–Feynman entre éléments de trajectoire est adoptée. Le 6-champ* B^{ij} *impliqué dans les forces et le 4-potentiel* A^i *impliqué dans les énergies et les impulsions sont considérés comme également physiques, du fait que la condition d'intégration est fixée via l'équivalence masse-énergie et l'opposition action-réaction. Ceci est tenu pour preuve par l'absurde que le photon a une masse propre. Comme exemple topique on discute l'interaction entre un aimant toroïdal lourd au repos dans le laboratoire et une charge ponctuelle en vol. Les connections avec l'effet Aharonov–Bohm et la diffraction électronique broglienne sont examinées. Des tests cruciaux de* l'énoncé fondamental *usant d'un aimant toroïdal sont proposés.*

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