*Foundations of Physics Letters, Vol. 13, No. 4, 2000*

# TIGHT MACH-STYLE CONNECTION BETWEEN THE EINSTEIN MASS-ENERGY AND INERTIA-GRAVITY EQUIVALANCES. REMARKS ON A RECENT EXPERIMENT BY MIKHAILOV

**O. Costa de Beauregard**

*Fondation Louis de Broglie, 23 rue Marsoulan*
*75012 Paris, France*

Received 15 December 1999; revised 8 January 2000

*Equivalence* of the two famous Einstein equivalences is, $U$ denoting the cosmological potential, expressed à la Mach as $U = c^2$. The electrostatic analog is induction of an extra mass $-c^{-2}eV$ (esu) in an electron immersed in the constant Coulomb potential $c = Q/R$ enclosed in a charged sphere. This effect has been evidenced in a recent experiment by Mikhailov.

Key words: Gravitational or electromagnetic gauges, integration conditions.

## 1. $c^2$ AS AN EXPRESSION OF THE COSMOLOGICAL GRAVITATION POTENTIAL $U$

Woodward and Mahood [1] in great detail, and I [2] tersely, have pointed out that Sciama's [3] updating of Berkeley's and Mach's conjecture regarding the nature and origin of inertia is expressed by the equality $U = c^2$ of the uniform background gravitational potential and the squared velocity of light. Compare the formulas $W_g := Um_g$ and $W_i := c^2m_i$, the first expressing the graviational potential energy of a test particle of passive gravitational mass $m_g$ and the second the energy equivalent to its inertial mass $m_i$. The equality $m_g = m_i$ was inferred from a thought experiment by Galileo. Assuming the equality $W_g = W_i$ is an updated form of Mach's conjecture. The equality $U = c^2$ then follows. This means that inertial mass is induced via gravitational potential energy, expressing an equivalence of the two Einstein equivalences. In other words, assertion of the equivalence of mass and energy, and the equivalence of inertial and passive gravitational mass, are equivalent if Mach's principle is true.

In Sciama's development, in a picture where $U$ is thought of as generated by 'the sphere of fixed stars' of mass $M$ and radius $R$, one gets the well known formula $GM/c^2R = 1$, $G$ denoting Newton's constant of gravitation. This entails a curious binding of the very large and the very small as $c^{-2}G$ also is the ratio of the Planck length $l$ and mass $m$: one has $c^{-2}G = R/M = \ell/m$. A rationalization of this may be afforded by Nottale's [4] *Theory of Scale Relativity.*

So the ominipresent effect named *inertia* evidences *gravitational action-reaction* between an accelerated body and *all* other bodies. As a consequence *the cosmological potential $U$ is a physical magnitude of value $c^2$; adding an arbitrary constant is strictly forbidden. Mutatis mutandis*, this must be true in electricity. An electron (for example) of mechanical mass $m$ and charge $-e$ evolving in the uniform fieldless Coulomb potential $V = Q/R$ enclosed in a charged sphere should be endowed with an *induced extra-mass* $-c^{-2}eV$ and exhibit an *effective inertial mass* $m - c^{-2}eV$ — thus evidencing the Coulomb potential as a measurable physical magnitude.

## 2. DIGRESSION ON A RECENT EXPERIMENT BY MIKHAILOV

It is well known but largely underestimated [5,6] that Einstein's *mass energy* equivalence entails that the weight of an electrically bound system comprises an 'electromagnetic mass defect.' Thus selected as an integration condition the Coulomb potential, widely thought to be adjustable by a gauge transformation of the first kind, truly is a *measurable magnitude*; there is a measurable contribution $\frac{1}{2}c^{-2}\Sigma VQ$ to the mass of an electrodynamically bound system. Relativistically speaking, however, there is a difference between the electrostatic $W_e = -eV_e$ and the gravistatic $W_g = m_gU$ formulas: the electric charge is a Lorentz invariant scalar whereas it is the gravity potential that is a scalar.

In a recent ingenious experiment Mikhailov [7] has shown that the inertial response of an electron accelerated inside the fieldless uniform Coulomb potential $V = Q/R$ enclosed in a charged sphere displays an effective mass $m - \frac{1}{2}c^{-2}eV$, meaning equipartition of the mutual energy in accord with the traditional writing. Discarding arguties on '1/2 or not 1/2' the emphasis is: *measurability of the fieldless Coulomb potential $V = Q/R$ enclosed in a charged sphere.* This claim had been issued previously, independently, by Assis [8] and I [5,6].



## 3. ELECTRODYNAMICALLY INDUCED LINEAR AND ANGULAR MOMENTA

By relativistic covariance selection as *an integration condition* of the electric gauge via energy-mass equivalence entails selection of the magnetic gauge via action-reaction. This is how a connection exists between Mikhailov's finding and the so-called [9,10] *hidden momentum in magnets or current loops* disclosed in three 1967 independent works [11,12,13].

The phenomenon consists of this. An Ampèrian circuit of intensity $i$ immersed in an electrostatic potential $V(r)$ and field $\mathbf{E}(\mathbf{r}) = \partial V(r)$ contains an *induced momentum*

$$\mathbf{p} = c^{-2}i\int \mathbf{V}\cdot d\mathbf{l} = c^{-2}i\int\int \mathbf{E}\times ds. \quad (1)$$

Via Ampère's equivalence rule, a small circuit of area $\mathbf{s}$ and magnetic moment $\mathbf{M}$ then contains an *induced linear momentum*

$$\mathbf{p} = c^{-2}\mathbf{E}\times\mathbf{M}. \quad (2)$$

Shocley's and James [13] very heuristic derivation is difficult to summarize. Penfield's and Haus [12] is Sommerfld style; it is via kinetic-plus-potential energy conservation of the conduction electrons and uses the velocity dependence of mass. My [15] derivation is via formula (2), the Coulomb model of magnetism, and the velocity dependent 'electric Lorentz force' felt by moving poles. Anyhow, *this is a relativistic $c^{-2}$ effect.*

Notwithstanding gauge invariance of formula (1) there is reason [5,6] to believe that the differential momentum $\mathbf{dp} = c^{-2}iVd\mathbf{l}$ has local meaning — as in Somererfeld's atomic theory. Linear momentum balance is the main reason [2,5,6]. If the potential $V$ is generated by a point charge $Q$ and expressed as $V = r^1Q$, by using the *source adhering* expression

$$\mathbf{A} = -i\int r^{-1}d\mathbf{l} \quad (4)$$

of the circuit's vector potential, *linear action-reaction* is formalized as $\pm\mathbf{p} = Q\mathbf{A}$; the differential momentum $\mathbf{dp} = c^{-2}iVdl$ then has a local meaning — and this is what Mikhailov finds. Then the circuit or magnet contains [2,5,6] both a linear and an angular induced momentum — *even in the fieldless potential enclosed in a charged sphere.* For a circuit its expression is

$$\mathbf{C} = c^{-2}i\int V\mathbf{r}\times dl; \quad (5)$$

for a very small circuit of area $s$ and magnetic moment $\mathbf{M} = is$ one has [5,6]

$$\mathbf{C} = 2c^{-2}V\mathbf{M}. \quad (6)$$

## 4. CONCLUDING

Mach's Principle, an attractive idea, has not been very productive. It motivated Einstein's quest for a basic explanation of gravity but found no place in general relativity. I submit that *where it has significance is in the interplay between gravity and any other physical field* — for example electromagnetism. This is because *any field interacts with gravity via inertia:* "any material point is a massive point."

Let it be recalled [14] that the link between Ampère's *Electrodynamics* and Maxwell's *Treatise* is mainly the valuable work of the German "electricians." They understood that $c$, the velocity of light in *in vacuo*, equals the ratio of the magnetic and electric expressions of charge, produced a static measurement of $c$, the telegraphing of oscillating signals, and the Gauss "absolute units." In Weber's electrodynamics [6] a particle of mechanical mass $m$ and (say) charge $-e$ exhibits a (possibly negative) *effective mass* $m - c^{-2}eV$.

This, of importance to us, can be evidenced [14] by an elementary calculation in the 'low velocity and instantaneous approximation.' Action-reaction between two particles $a$ and $b$ starting from rest consists of the Coulomb force *plus* the induced electric force due to acceleration of the other particle. If, say, particle $b$ is replaced by a charged sphere enclosing particle $a$ the mutual Coulomb force disappears *but the reciprocal forces induced by mutual acceleration remains.* Accelerated *by any means* inside the charged sphere, the electron imparts to it, via the induced electric field, a force *and certainly feels the reaction* which comes out as $(m - c^{-2}eV)\ddot{\mathbf{r}}$.

Heaviside did go too far in "assassinating" the electromagnetic potentials. Of course the electromagnetic *forces* depend on the *fields* — but their integrals over space *energies* or time *linear and angular mementa* do depend on the *potentials.* In a bound system, *they have definite expressions fixed as integration conditions.* These express *mass-energy equivalence* and *linear and angular momentum balance.* Namely, *they evidence the interplay between gravity and the other field.*

**Acknowledgements.** Helpful comments by two anonymous referees are gratefully acknowledged.

## REFERENCES

1. J. F. Woodward and T. Mahood, *Found Phys.* **29**, 899-930 (1999).
2. O. Costa de Beauregard, *Ann. Fond. Louis de Broglie* **23**, 135 (1999).
3. D. W. Sciama, *Month. Not. Roy. Astr. Soc.* **113**, 34-42 (1953).
4. L. Nottale, *Fractal Spacetime and Microphysics. Scale Relativity* (World Scientific, Singapore, 1993).



5. O. Costa de Beauregard, "Electrodynamic gauge as an integration condition," in *Advanced Electrodynamics*, R. W. Barrett and D. M. Grimes, eds. (World Scientific, Singapore, 1995), pp. 77-104.
6. O. Costa de Beauregard, *Phys. Essays* **10**, 492 and 646 (1997).
7. V. F. Mikhailov, *Ann. Fond. Louis de Broglie* **24**, in press.
8. A. K. T. Assis, *Weber's Electrodynamics* (Kluwer Academic, Dordrecht, 1994), pp. 193-198.
9. S. Coleman and J. D. van Vleck, *Phys. Rev.* **171**, 1370 (1970).
10. A. S. Goldhaber and W. P. Trower, *Am. J. Phys.* **58**, 429 (1990).
11. O. Costa de Beauregard, *Phys. Lett. A* **24**, 177 (1967).
12. P. Penfield and H. Haus, *Electrodynamics of Moving Media* (M.I.T. Press, Cambridge, Mass., 1967), pp. 202 ff.
13. W. Shockley and R. P. James, *Phys. Rev. Lett.* **18**, 876 (1967).
14. O. Costa de Beauregard, *Ann. Fond. L. de Broglie* **21**, 265 (1996); see p. 267.