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THE GOILLOT EFFECT AS TRANSLATIONAL INERTIAL SPIN EFFECT

O. COSTA DE BEAUREGARD
Institut Henri Poincaré, Paris

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A new electromagnetic effect is deduced, qualitatively similar to Goillot's, but much smaller, it is thus concluded that Goillot's effect is dynamical rather than electromagnetic, and implies, as in many spin theories, a non-collinearity of the momentum and velocity 4-vectors.

Considering, 1) the observed features of the new effect experimentally demonstrated by Goillot [1], 2) vector and dimension analysis, there are only two possible formulas for the kinetic momentum ΔL imparted to the test body:

$$\Delta L = k_1 c^{-1} \Delta \iiint D \times B \,\mathrm{d}v, \qquad (1)$$

$$\Delta L = k_2 \Delta \iint \sigma \times s. \qquad (2)$$

In (1), $D(D_r, 0, 0)$ denotes a constant radial electric induction and $B(0, B_\theta, 0)$ a variable tangential magnetic induction in mixed Heaviside units; in (2), $\sigma(0, \sigma_\theta, 0)$ denotes a variable tangential spin density, the surface element ds being radial on the cylindrical internal and external surfaces of the test body; k_1 and k_2 are dimensionless coefficients, and c the velocity of light. Thus, in both cases, $\Delta L(0, 0, \Delta L_z)$ is axial, with a sign depending on that of ΔB or $\Delta \sigma$, as is observed.

First we show that Goillot's test body may well exhibit a recoil effect of expression (1) which : 1) is a new electromagnetic effect, that is, a yet undeduced consequence of Maxwell's theory; 2) is a new direct proof that the asymmetric electromagnetic energy momentum tensor as defined by Maxwell and Minkowski is operationally the good one*; 3) is numerically smaller than Goillot's observed effect by a factor of order 10^{-8}.

If the axial wire [1] has a permanent uniform charge q per length unit, it will receive, when the magnetic polarization $M \equiv B - H$ is reversed inside the test body, a momentum of value

$$\Delta L_z = -c^{-1} q \int_{-\infty}^{+\infty} (\Delta A.\, u)\,\mathrm{d}z \qquad (3)$$

where A denotes the vector potential and u the unit vector along the z axis. Applying the induction flux formula to a closed contour comprising the rectilinear wire and a semi-circle with infinite radius and center at the center of the test body yields

$$\Delta P_z = -c^{-1} q \Delta\Phi \approx -c^{-1} q\, a\, b\, \Delta M, \qquad (4)$$

with a denoting the height and b the difference between the internal and external radii of the test body.

The conservation of momentum requires that an opposite momentum appears either in free space or inside the test body. The only possibility is the latter one, the corresponding expression being (1) with $k_1 = 1$, and the electric field

* A long controversy has opposed to the proponents of the asymmetric electromagnetic energy momentum tensor (Maxwell, Heaviside, Minkowski, Nordström) the partisans of the symmetrized tensor (Hertz, Abraham, Grammel, Pauli). Precise arguments in favor of the asymmetric tensor due are to Tamm and von Laue. For a résumé of the discussions and references see ref. 2.

$E_\gamma = q/2\pi r$ generated by the charged wire inserted as D_γ.

Such a recoil effect of the test body directly prooves that the asymmetric Maxwell-Minkowski tensor is the "good" one. Minkowski [3] has demonstrated the formula $(i, j, k, l, = 1, 2, 3, 4; x^4 = ict)$

$$f^i = B^{ik} j_k + \tfrac{1}{4}\,[B^{kl}\partial^i H_{kl} - H_{kl}\partial^i B^{kl}] = \partial_j\,\{-B^{ik} H^j_k + \tfrac{1}{4} B^{kl} H_{kl}\,\delta^{ij}\}, \tag{5}$$

where B^{ij} denotes the magnetic induction-electric field, H^{ij} the magnetic field-electric induction, and j^k the 4-current density; the tensor $\{^{ij}\}$ is the Maxwell-Minkowski tensor. The total 4-force density is thus written as the sum of the Lorentz one plus the "Stern-Gerlach" one (force density applied to polarized bodies by inhomogeneous fields).

Integrating (5) on the 4-dimensional space-time volume enclosed between an initial and a final state of an infinitely heavy test body, and the timelike "wall" generated by the contour of the body, yields

$$\iiiint \boldsymbol{f}\,\mathrm{d}v\,\mathrm{d}t - \iiint\{(\boldsymbol{D}.\,\mathrm{d}s)\boldsymbol{E} + (\boldsymbol{B}.\,\mathrm{d}s)\boldsymbol{H} + - \tfrac{1}{2}(\boldsymbol{E}.\boldsymbol{D} + \boldsymbol{H}.\boldsymbol{B})\mathrm{d}s\}\,\mathrm{d}t = c^{-1}\,\Delta\iiint \boldsymbol{D}\times\boldsymbol{B}\,\mathrm{d}v, \tag{6}$$

$$\iiiint f_4\,\mathrm{d}v\,\mathrm{d}t - c\iiint(\boldsymbol{E}\times\boldsymbol{H})\,\mathrm{d}s\,\mathrm{d}t = \Delta\iiint \tfrac{1}{2}\,(\boldsymbol{E}.\boldsymbol{D} + \boldsymbol{H}.\boldsymbol{B})\,\mathrm{d}v. \tag{7}$$

According to these formulas, $c^{-1}\boldsymbol{D}\times\boldsymbol{B}$ is the momentum density and $c\,\boldsymbol{E}\times\boldsymbol{H}$ the energy current density of the field. Incidentally, according to (6), the driving force applied to Goillot's test body is the surface force $-(\boldsymbol{D}.\,\mathrm{d}s)\boldsymbol{E}$, with $D_\gamma = q/2\pi r$ and $E_\theta = -\partial A_z/c\partial t$ as before.

The new electromagnetic effect just described has the qualitative features of the Goillot effect. But, when calculating the premanent* charge q per unit length of the wire and the associated potential difference wire and test body, which would correspond to the observed effect, one finds a value some 10^4 times too strong.

Thus, formula (1) is excluded, and we are left with formula (2) as the only possible explanation of the Goillot effect. Such a formula, with $k_2 = 1$, was in fact the one that we [4] had initially put forward, and it is for testing it that Goillot devised his experiment. It had been rightly objected [5] to our deduction that, calculating the surface integral (2) just outside rather than just inside the test body would yield $k_2 = 0$ rather than $k_2 = 1$. In fact, Goillot's experiment yields very definitely $|k_2| \approx 0.03$.

In our opinion, the conclusions derived from experimental facts should be : 1) that Goillot's spin containing test body has in effect a momentum non collinear to its velocity, as is the case in numerous spinning particles theories; 2) that our initial theory was oversimplified, and that the intricate collective theory of electrons inside ferromagnetic solids may leave room** for the possibility that the overall momentum is not collinear with the velocity of the solid.

References

1. C.Goillot, Physics Letters 21 (1961) 408.
2. C.Möller, The theory of relativity (Clarendon Press, Oxford, 1957) p. 202-206 and W.Pauli, Theory of relativity (Pergamon Press London, 1958) p.108-111 and note 11 p.216.
3. H.Minkowski, Math. Ann.68 (1910) 472; see formula (91) p. 511.
4. O.Costa de Beauregard, Phys. Rev. 129 (1963) 466. See formulas (33) and (34) p. 469.
5. A.Papapetrou, J.S.Bell, B.d'Espagnat and F. Halbwach, private communications.

* If the signal reversing the magnetization of the test body is in the form of a propagating rather than standing wave, it will: 1) charge the wire (either negatively or positively) and thus induce a recoil effect as shown in the text; 2) create a field gradient $\partial_z\boldsymbol{H}$ and thus a "Stern Gerlach" force density $\boldsymbol{M}.\partial_z\boldsymbol{H}$. It is easily verified that these two perturbing effects exactly compensate each other, so we are left only with the effect of a *permanent* charge of the wire, as discussed in the text.

** Noether's theorems which entail the formula $T^{ij} - T^{ji} = \partial_k\,\sigma^{ijk}$ used in (5), do not say anything about velocities.

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