No Paradox in the Theory of Time Anisotropy*

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Summary. Factlike rather than lawlike character of the physical irreversibility principle expressed by means of Bayes' principle, and considered as a boundary condition for integrating the macroscopic evolution equations. The Ritz-Einstein controversy reexamined and J. von Neumann's irreversibility of quantum measurements reworded: one to one connection between the principles of statistical irreversibility and of wave retardation in the theory of quantized waves. Factlike time asymmetry and lawlike time symmetry in the cybernetic context.

Before we formulate (in accord with quite a few recent authors) the answer we feel appropriate to the question in the title, it is useful to understand why it has been felt that the theory of the time anisotropy contains a paradox.

1. The Paradox behind the Loschmidt and Zermelo Paradoxes: Time Anisotropy in the "Principle of Probability of Causes"

The well known Loschmidt and Zermelo "paradoxes" in statistical mechanics have merely uncovered the existence of a much older "paradox" inherent in the probability theory itself since the early days of Pascal, Fermat and Bayes, where it came to be named, very significantly, the "principle of probability of *causes*". That is to say, it was obscurely felt that the time anisotropy inherent in the anthropomorphic notion of a "cause" developing aftereffects rather than before effects is somehow connected with the empirical fact that, *even if the transition probabilities between two possible states of a system are symmetric* (as in such classical examples as card shuffling) more probable macroscopic complexions follow in time less probable ones – not the other way.

As Watanabe puts it, the *empirical fact* is that *blind statistical prediction is physical while blind statistical retrodiction is not* – a situation with which probability theory copes by using in retrodictive problems Bayes' formula for conditional probability. But, as the Bayes coefficients are by definition independent of the internal dynamics of the system under study, this amounts to

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saying that the theoretical description of time anisotropy in probability problems is of an extrinsic rather than intrinsic nature. It is, very exactly, a *boundary condition* imposed upon the macroscopic evolution equations. This boundary condition reads "blind retrodiction forbidden", very much like the boundary condition in macroscopic wave theories reads "advanced waves forbidden". The point is that in both cases the boundary condition is an initial, not a final condition. To this we will come back later.

Now we stress the connection between the temporal application of Bayes' principle and the causality concept. To say that the evaluation of Baye's coefficients is extrinsic to the dynámics of the system is to say that they are used to describe an interaction between the system and its surroundings. And to say that the Bayes coefficients must in fact be used in retrodictive but not in predictive problems amounts to saying that the effects of the interaction upon the system are felt after it has ceased, and not before it has begun. But *this* is the very definition of causality, that is, "retarded actions", as opposed to finality or "advanced actions".

A typical example of this general physical law is the ink drop that dissolves in a glassful of water after it has been deposited in it by a pipette; the reversed procedure (including the pipette and the hand holding it) can only be seen by running backwards a movie film. The same can be said of card shuffling[1] or radioactive disintegration.

We thus come to the conclusion that *the time dissymetry inherent in causality as opposed to finality is of an essentially macroscopic nature,* and that its mathematical formulation consists in the temporal application of Bayes principle expressing that blind statistical retrodiction is forbidden *in physics.* To our cognizance Van der Waals was the first, in 1911, to state that the statistical derivation of Carnot's law is merely a temporal application of Bayes' principle – an idea which is also implicit in an often quoted sentence of Willard Gibbs (1914)[2]. That the mathematical expression of statistical irreversibility is, as Mehlberg (1961) puts it, of a factlike rather than lawlike character has also been expressed in recent years by Watanabe, Reichenbach, E. N. Adams, J. A. Mc Lennan, Wu – Rivier, Grünbaum, C. F. von Weizsäcker, G. Ludwig, O. Costa de Beauregard. The identification of the causality concept with the physical law of increasing probabilities has been especially strongly stressed by Reichenbach, Grünbaum, Terletsky, Costa de Beauregard.

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1 Of course a deck of cards is said to be "in order" or "in disorder" according to the fact that the sequence of the cards belongs or not to some selected small sub-ensemble of the ensemble of possible permutations of the cards.

2 "It should not be forgotten, when our ensembles are chosen to illustrate the probabilities of events in the real world, that while the probabilities of subsequent events may often be determined from those of prior events, it is rarely the case that probabilities of prior events can be determined from those of subsequent events, for we are rarely justified in excluding the consideration of the anteceding probability of the prior events".

2. *The Einstein – Ritz Controversy: Retarded Waves and Probability Increase*

The existence of a close connection between the two principles of wave retardation and probability increase is strongly suggested by many physical examples such as, for instance, the slowing down of a meteorite in the earth's atmosphere. In these recent years it has been more or less explicitly stated, in various contexts, by quite a few authors among those listed at the end of this paper (Mc Lennan, Penrose – Percival, Costa de Beauregard and others). To our cognizance the discussion started in the late 1900's with the celebrated Einstein-Ritz controversy, in which Ritz insisted for deducing the law of entropy increase from the principle of wave retardation while Einstein maintained that the law of wave retardation should follow from the principle of probability increase.

That Einstein's and Ritz' statements are *reciprocal* should be obvious now that the formulation of the principle of statistical irreversibility has been recognized to be of the nature of a boundary condition.

The aim of this Section is to show that, in a restricted but precise context, wave retardation and probability increase are indeed two names for one and the same principle. At this end we will work with a theory implying *essentially* the two concepts of waves and probability, namely, quantum mechanics.[3]

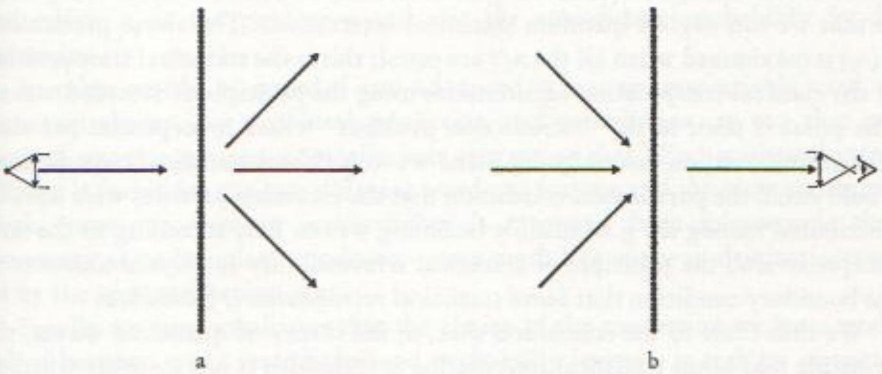

Fig. 1 a and b. Physical Irreversibility as a Boundary Condition: the quantized wave and grating thought experiment. (a) Retarded waves and blind statistical prediction. (b) Advanced waves and blind statistical retrodiction

3 The connection between the two principles of probability increase and wave retardation is implied in Planck's definition of the entropy of a light beam, where the constant h, that is, the photon concept, is essential. It follows from Planck's formula that the entropy of a light beam is increased by scattering or "dif-fusion", a time asymmetric process following from the principle of retarded waves. If advanced waves were macroscopically existent, then phase coherent "in-fusion" would decrease the entropy of the light beam. Of course, in the days of the Einstein-Ritz controversy, the photon concept had already "come to light", but de

Let us first illustrate our statement by using an example. A plane monochromatic wave falling upon a linear grating (the wave planes being parallel to the lines on the grating, which we assume infinite in number for simplicity) generates a finite number g of outgoing plane monochromatic waves; this follows from the necessity of phase coherence and the principle of retarded waves. Now, if one of these outgoing plane waves is received in a collimator and the observer knows nothing else than the presence of the grating (and, of course, the space frequencies of both the wave and the grating), the only thing he can say is that an incoming plane wave falls on the grating and that it is one among a well defined class of g waves (comprising the one considered first). He does *not* conclude that the wave he receives is built up by phase coherence of the g possible incident waves, because this would amount to accept the macroscopic existence of advanced waves.

Now we must remember that our waves are assumed to be quantized, so that we can transpose a discourse on intensities into a discourse on probabilities. If, in a predictive problem, n corpuscles with the same sharply defined momentum fall per time unit upon the grating, then, to use Watanabe's excellent terminology, a "blind statistical prediction" yields $n!/\pi\ (n_i!)$ as the probability $p\ (n_i)$ that n_i corpuscles per time unit come out on each of the admissible outgoing waves; for simplicity we have assumed that the transition probabilities between the g initial and the g final mutually exclusive states are equal; also, that n is low enough for each wave train to carry not more than one corpuscle, so that we can neglect quantum statistical interactions. The above probability $p\,(n_i)$ is maximized when all the n_i's are equal; this is the statistical transposition of the classical computation of intensities using the principle of retarded waves. The point is that, in the "retrodiction problem" where n corpuscles per time unit are received on one outgoing plane wave, a "blind statistical retrodiction" would entail the paradoxical conclusion that the incoming particles were equally distributed among the g admissible incoming waves. But, according to the now accepted view, the principle of statistical irreversibility in physics amounts to the boundary condition that *blind statistical retrodiction is forbidden*.

We thus come to the conclusion that, in the theory of quantized waves, the principle that blind statistical retrodiction is forbidden is just an other wording for the principle that advanced waves are *macroscopically* non existent.

The consideration of this (or any similar) example is a good preparation for understanding the abstract proof of the equivalence in quantum mechanics of the two irreversibility principles that will now be presented. This proof merely consists in a rewording of J. von Neumann's celebrated theorem on statistical irreversibility in the measuring process.

Broglie's symmetrical association of waves with matter particles was still lying in the future. Had Einstein and Ritz known that particle and wave scattering go hand in hand, then both of them would have recognized that they were merely issuing *reciprocal* statements.

In a simplified form von Neumann's irreversibility statement boils down to this. Denoting p_{ij} the (intrinsic) transition probabilities between some initial $|i\rangle$ and some final $|j\rangle$ orthonormalized set, p_i and p_j the (extrinsic) statistical weights of the $|i\rangle$'s and the $|j\rangle$'s, the formula

$$p_j = \sum_i p_i \, p_{ij}$$

holds in a predictive calculation. Then, denoting p the largest p_i and using the normalization condition

$$\sum_i p_{ij} = 1$$

the result

$$p_j \leq p \text{ for all } j\text{'s}$$

follows; this is a typical instance of the levelling of statistical frequencies in blind prediction, that is, physical prediction.

The point is that the principle of retarded waves has been implicitly used when stating that, *macroscopically speaking in the sense of J. von Neumann's ensembles,* p_j is the predictive and *not the retrodictive* probability in the transition $|i\rangle \rightleftharpoons |j\rangle$.

In other words, as retarded and advanced waves are respectively used, in quantum theory, for statistical prediction and retrodiction, to say that advanced waves are *macroscopically* non existent or that blind statistical retrodiction is forbidden are two different wordings for one and the same statement. Both ways, macroscopic irreversibility is extracted from microscopic time symmetry *via* a boundary condition – very much like one way driving is secured by the appropriate sign post.

Finally we must emphasize that the clarity of the connection we have established between wave retardation and probability increase is such by virtue of a precise, but narrow context. J. von Neumann's micro-entropy (which, for brevity, has not been introduced explicitly) is a much simpler concept than the macro-entropy of thermodynamics. Nevertheless, as we have said, there are so numerous examples of an observable connection between wave retardation and entropy increase that I am confident that this kind of argumentation can be very largely extended. For instance, if between time instants t_1 and t_2 a physicist moves a piston in the wall of a vessel containing a gas in equilibrium, the fact is that Maxwell's velocity distribution law is alterated after time t_2, not before time t_1. But the fact is also that the perturbation is emitted (not absorbed) by the moving piston as a retarded (not advanced) wave.

3. *The Factlike Character of Physical Irreversibility and the Information Concept*

It is certainly very striking that cybernetics has rediscovered, without having searched for it, the twofold aspect of the old Aristotelian "information" concept, namely (i) gain in knowledge and (ii) organizing power. That (except in a few philosophical circles interested in finality) the second aspect of Aristotle's "information" happened to be almost completely forgotton can be understood as a consequence of physical irreversibility, as will now be explained.

That information is a two faced concept in cybernetics is quite obvious in the characteristic chain.

$$information_1 \rightarrow negentropy \rightarrow information_2$$

of communication systems or computers; also, in the characteristic chain

$$negentropy_1 \rightarrow information \rightarrow negentropy_2$$

of physical measurements and the classifications they allow such as, for example, in the Maxwell demon problem analyzed by Smoluchowsky, Szilard, Demers, Brillouin, or the von Neumann measuring process in quantum mechanics. The *learning transition*

(i) *negentropy* → *information*

appears there as symmetrical to the *acting transition*

(ii) *information* → *negentropy*.

In (i) *observational awareness* follows in time the physical situation which, in accordance with Reichenbach's analyses, it *registers*. In (ii) *willing awareness* precedes in time the physical situation which it contributes to *produce*.

Physical irreversibility consists in the *fact* that the above arrows all point towards lower information or negentropy values. Thus, for instance, the learning transition (i) appears as a generalization of the passive Carnot degradation of negentropy in closed systems. But, as Mehlberg and others have so strongly stressed, physical irreversibility is of a factlike rather than lawlike character. Thus cybernetics implies an invitation to inquire about the lawlike rather than factlike status of our problem – very much like the internal symmetries of the Dirac electron theory have been justified by the discovery of Anderson's positron which, though *de facto* much rarer than the electron, is *de jure* its twin brother.

The irreversibility principle, as stated in the two preceding Sections, amounts to saying that, physically speaking, a low probability complexion can *in fact* be taken as the starting point of a regressing fluctuation rather than the end point of a progressing fluctuation. So the factlike (not lawlike!) irreversibility principle of cybernetics turns out to be that the learning transitions (i) are more frequent than the acting transitions (ii). In terms of awareness, observation is easier, or less tiring, than action.

Now, it should be quite clear that the very value of universal constants in terms of "practical" or anthropomorphic physical units reflects an existential situation. For instance, to say that the value of the light velocity c is "very large" is to say that the ratio of associated length and time units we find convenient is much smaller; this fortuitous circumstance (which, in our opinion, may well stem from the very value of our nervous influx velocity: some small multiple of 1 m/sec) is, as is now well known, at the origin of everybody's (wrong) feeling that "there is an absolute time". Quite similarly, the "smallness" of Boltzmann's constant k and of the conversion coefficient k Ln 2 between an entropy expressed in "practical" thermodynamic units and an information expressed in binary units, according to the equivalence formula

$$\textit{negentropy} = k \operatorname{Ln} 2 \times \textit{information},$$

may well be taken as a direct expression of the *fact* that observation is for us much easier than action: the conversion rate is such that gaining knowledge is very cheap in negentropy units while producing negentropy costs a lot in *bits*. Going to the limit $k \to 0$ would imply that observation is completely costless and action impossible. This crude approximation to cybernetics has been known as the theory of "epiphenomenal consciousness".

Our final remark will be almost philosophic. Since long ago it has been recognized that progressing fluctuations and advanced waves can be taken as the objective aspect of finality, just as regressing fluctuations and retarded waves are now understood to be *the* physical expression of causality. There should then be no wonder that the finality concept is so elusive in terms of cognitive awareness, for it follows from above that the learning transition being "causal" and the acting transition being "final" (in the above sense), the evidence of causality belongs to cognitive awareness just as the evidence of finality belongs to willing awareness. This, of course, is well known to philosophers, but cybernetics helps understanding why things are so.

Conclusion

We have resumed in modern terms the old "paradoxical" problem of deducing physical irreversibility from elementary laws assumed to be time symmetric. We have found, with quite a few recent writers, that strictly speaking there is no paradox at all, but merely a *factlike* state of affairs which is mathematically expressible as an appropriate boundary condition.

As for the lawlike status existing beyond the factlike situation we feel that cybernetics has something to say. Very strikingly, a quantum measurement essentially implies a perturbation on the measured system so that, in this sense, cognizance and action are inseparable. More thinking and more knowledge are needed here, and we feel that quantum biology might well take part in the discussion.

Finally there is of course the possibility that the recent *PC* violations in elementary particle physics imply *T* violations that should be superposed as a slight perturbation upon the preceding scheme. But this is part of to-morrows problems.

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